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Application of
Multiple Lines of Position
for Hydrography

by

Samuel P. De Bow, Jr.

March 1986

Thesis Advisors: Narendra K. Saxena
Glen R. Schaefer

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Application of Multiple Lines of Position for Hydrography

by

Samuel P. De Bow, Jr.
Lieutenant, National Oceanic and Atmospheric Administration
B.S., Drexel University, 1976

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Author: Samuel P. De Bow, Jr.
Samuel P. De Bow, Jr.

Approved by: Narendra K. Saxena
Narendra K. Saxena, Co-Advisor

Glen R. Schaefer
Glen R. Schaefer, Co-Advisor

Christopher N. K. Mooers
Christopher N. K. Mooers, Chairman,
Department of Oceanography

John N. Dyer
John N. Dyer,
Dean of Science and Engineering

ABSTRACT

Conventional near-shore large-scale hydrographic surveys use only two lines of position (LOPs) for position fixing. Previous works have proved that accuracies needed to meet the present International Hydrographic Organization standards are frequently not achieved using conventional surveying methods. The concept of using multiple lines of position (MLOP) adjusted by the least squares method was described. Actual field measurements acquired in the autumn of 1984 were processed to ascertain the increase in accuracy using MLOP versus conventional two LOPs on each hydrographic position. Recommendations are to use a four-range fully automated position-fixing method to increase production, improve data quality, and have better control of the survey operations. Keywords: → (p. 1)

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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. BACKGROUND

A hydrographic survey is a compilation of numerous types of data which are acquired to ultimately produce a nautical chart. The two most important aspects of a hydrographic survey are (1) the depth sounding of the seabed and (2) the simultaneous acquisition of a horizontal position which is attached to the sounding. If either of these two aspects are in error then the overall accuracy of the survey is lost. This thesis will address the latter aspect. Ingham [1984, p. 14] states

> A major element of the hydrographic survey is the dynamic positional control of the survey vessel. Indeed, the great majority of the surveyor's tasks offshore today require positioning services alone.

The accurate determination of a position at sea has always been a painstaking effort for the hydrographic surveyor. First, a survey of the coast adjoining the offshore survey area must be made to establish an accurate position of each shore control site. This shore control must then be "extended" offshore by a variety of means and systems depending on the scale and purpose of the survey. The position of a survey vessel is either determined by a visual method, such as a three-point sextant fix to visual

signals onshore, or by a radio navigation method. In the very near future, the Global Positioning System (GPS) promises to completely revise the method of offshore navigation and surveying. Consistent differential positional accuracies of about 10 m, or less, are expected in real time once the system becomes fully operational thus allowing the hydrographer the freedom of surveying as far offshore as the situation demands.

This thesis will address various ways to increase the positional accuracy of nearshore surveys. Nearshore areas (i.e., channels, harbors, and bays) pose the most hazard to the mariner and are the areas where accurate position determination of the bottom features is essential. Nearshore surveys of 1:5,000 scale, and larger, require intensive and time consuming delineation of shoreline features and bottom topography with positional accuracies of 5 m, or less. Presently, GPS will not provide these accuracies in a real-time mode. Conventional hydrographic survey methods are the only way to determine the position of a survey vessel in real time. Mobley stated [Saxena, 1980, p. A-4],

> Positioning systems for precise navigation for survey operations are barely adequate for the scales of surveys being accomplished today. A lot of time and effort is lost due to equipment down time, moving equipment and the use of only 2 lines of position . . . . which the present National Ocean Survey (NOS) DAS can only accept. New and future replacement systems providing redundant LOPs are a must if higher accuracies are desired

offshore. Global Navigation may be the answer but will be 5 to 10 years before readily available to all users.

If conventional systems and methods are to be used, a redundancy in the measurements is necessary to obtain a higher order of accuracy than the present system of acquiring two lines of position (LOPs) on each fix. This thesis will investigate the application of multiple lines of position (MLOP), adjusted by the least squares method, for hydrographic position control.

B. HYDROGRAPHIC STANDARDS OF ACCURACY AND METHODS

The National Ocean Service (NOS) is the agency responsible for nautical charting in the United States and has documented many standards and procedures for the acquisition of data during a hydrographic survey. With respect to the position of the survey vessel, the Hydrographic Manual [Umbach, 1976, p. 4-14] states

> Hydrographic position fixes on sounding lines are almost always determined by the intersection of two lines of position; dead-reckoning positions based on course and speed provide an additional internal check.

Also Ingham [1984, p. 15] states in his chapter on Position-fixing Afloat

> For any fix one requires at least two lines of position . . . obtained from readily identifiable sources. These should intersect at the largest angle possible for good fix-geometry, and have the smallest possible standard deviation, resulting in the smallest area of uncertainty.

NOS requires that for surveys of 1:5,000 scale, and larger, at least one of the LOPs must be from a visual method. Thus, a standard way of acquiring positional fixes on large-scale surveys has been the range-azimuth (rho-theta) method as addressed by Waltz [1983]. One of the reasons this hybrid method is popular is because the two LOPs will always intersect at 90°, the best intersection possible for good accuracy.

Unlike mariners, the hydrographer has access to more sophisticated and more accurate positioning systems. But as Riemersma [1980, p. 10] observes

> There is no mariner or surveyor in the world who in conventional navigation does not use a check bearing just to make sure. Why then does one, with radiopositioning [sic] systems, use only two LOPs?

By using only two LOPs there exists a level of uncertainty with each position which can only be resolved during the post processing phase of the survey [Perugini, 1984]. In the past, two LOPs were the most feasible and sometimes the only way to determine the position of the survey vessel. With the rapid advance in electronic technology, the acquisition of multiple-range LOPs are as common today as two-range LOPs were 20 years ago. However, due to cost limitations, or possibly the aversion of changing tried and true operational procedures, hydrographic surveys are still usually controlled with only two LOPs. The acquisition of

only two LOPs on each fix will change in the very near future for NOS as they are in the process of implementing a specification from the 1982 International Hydrographic Organization (IHO) [Anonymous, 1982, p. 5] stating

> It is desireable that whenever positions are determined by the intersection of lines of position, three such lines be used. The angle between any pair should not be less than 30°.

With MLOP the hydrographer will have redundant observations on each fix. These redundant measurements may be adjusted, as in normal land surveying, by the least squares method to acquire the "best" fit. Applying the least squares method to hydrographic position adjustment allows the hydrographer to statistically determine errors in real time if computer programs are operated aboard the survey vessel [Silva, 1982, p. 100].

C. OBJECTIVES

Will the use of MLOP increase the accuracy of the fix over the present method of two LOPs? This question has been addressed by both Kaplan [1980] and Silva [1982] who found that MLOP do improve the accuracy of the survey. However, the method was applied to theoretical data only. Therefore, one objective of this thesis was to acquire data for a portion of an actual hydrographic survey with a minimum of four LOPs on each position fix. A model was constructed to

determine the least squares position for each fix from the MLOP. The precision of each MLOP fix was compared to the precision of a fix obtained by two LOPs to determine if a major difference did exist between the two methods.

To obtain the additional LOPs, a significant increase in the cost of operations can be expected and must be justified. More LOPs will require more horizontal control for shore reference points. Additional survey equipment, electronic range measurement units or azimuth measurement units, or both, will be necessary to acquire the four LOPs. Initial equipment costs could possibly double at the onset of this method of surveying. Consequently, all of the costs associated with the MLOP method must be weighted against the optimal position accuracy requirement to determine if the additional accuracy is feasible within the scope of a normal hydrographic survey.

A final objective of this thesis was to determine an optimal system which is the most cost effective as well as one which will meet the new IHO standards for position fixing of soundings.

II. PROBLEM DEFINITION

The geometry of a standard position fix using two LOPs must be understood before the various MLOP methods may be evaluated. Therefore, the configuration of large-scale positioning methods, the errors unique to the equipment used, and the propagation of these errors into the offshore position warrant discussion in this chapter.

A. LARGE-SCALE SURVEY POSITIONING METHODS

The intersection of two lines defines a point in space. In navigation and offshore surveying each of the two lines, or LOPs, can be generated via an electronic or visual method. Traditional hydrography requires that the two LOPs generated intersect at an angle greater than 30° and less than 150°.

Notations used for computations are:

a) X coordinate positive to the east, Y coordinate positive to the north

b) Occupied Shore Stations; numerals 1, 2, . . .

c) Unoccupied Shore Station used for Initial Azimuths; numeral 0

d) Inverse Distance between shore stations; D

e) Ranges to vessel, point P; R_1, R_2, . . .

f) Computed horizontal angle; α

g) Measured horizontal angles; α_1, α_2, . . .

h) Azimuths of lines; $\alpha_{1,2}$, $\alpha_{1,P}$, . . .

1. Range-Range Positioning Method

An electronic range-range positioning method on large-scale surveys is generally accomplished with short-range (line-of-sight) equipment. At these short distances the ranges to the vessel are determined by measuring the time of transit or difference in transit times of an electromagnetic pulse [Davis et al., 1981]. The transit-time measurement starts with an interrogation pulse from a master transponder onboard the vessel which is received by a remote transponder erected over a known control point and returned to the master transponder. The total transit time is then converted to a distance by assuming an average value for the propagation velocity of the signal. The LOPs generated from a system such as Del Norte Trisponder (used for this thesis) are concentric circles from the shore stations. The Del Norte system operates at a frequency of 9 GHz with a pulse repetition interval (PRI) which can be selected from 304 to 998 microseconds. Thirty-two consecutive valid time-difference measurements are processed to provide each range output [Ingham, 1984, pp. 37-38].

The geometry of a range-range position fix using two shore stations is graphically illustrated in Figure 2.1. Basic trigonometric relationships are employed to determine the position of the survey vessel at point P using the

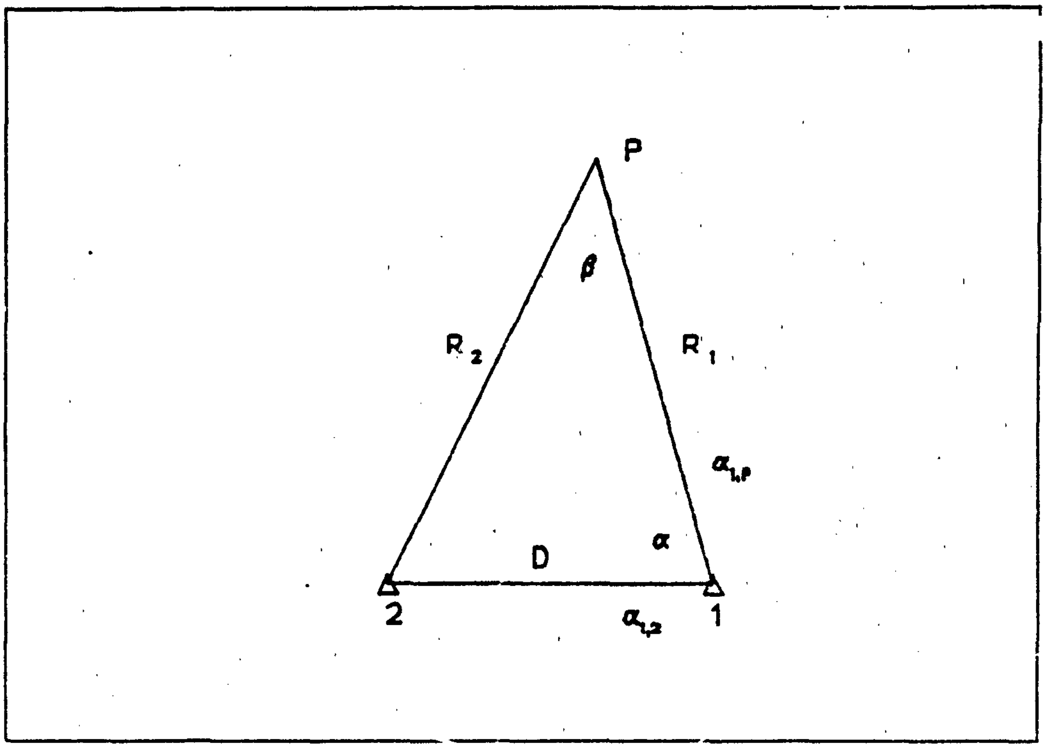

Figure 2.1 Range-Range Positioning Method.

measured ranges R_1 and R_2 from shore control stations 1 and 2, respectively.

The computation of the coordinates of the survey vessel, X_P and Y_P, at point P is as follows [Davis et al., 1981, p. 876]:

$$D = \sqrt{(X_1-X_2)^2 + (Y_1-Y_2)^2} \tag{2.1}$$

where X_1, Y_1 are the coordinates of shore station 1 and X_2, Y_2 are the coordinates of shore station 2. Since all three sides of the triangle are known, angle α can be computed from the law of cosines equation

$$\alpha = \text{arc cos } [(D^2 + R_1{}^2 - R_2{}^2)/(2\ R_1\ D)] \qquad (2.2)$$

The azimuth of the line from station 1 to station 2 can be determined by the inverse computation equation

$$\alpha_{1,2} = \text{arc tan } [(X_2 - X_1) \ / \ (Y_2 - Y_1)] \qquad (2.3)$$

The azimuth from shore station 1 to the vessel P is

$$\alpha_{1,P} = \alpha_{1,2} + \alpha \qquad (2.4)$$

If X_1 and Y_1 are the known coordinates of the left-hand station when looking from the vessel toward the base line, then the coordinates of the vessel at P can be computed by the equations

$$X_P = R_1 \sin (\alpha_{1,P}) + X_1 \qquad (2.5)$$

$$Y_P = R_1 \cos (\alpha_{1,P}) + Y_1 \qquad (2.6)$$

and the angle of intersection of the two range LOPs can be computed by the equation

$$\beta = 180° - \text{arc cos}[(D^2 - R_1{}^2 - R_2{}^2) \ / \ (2\ R_1\ R_2)] \qquad (2.7)$$

Determining the position of a vessel using multiple ranges from three, or more, shore stations will involve additional modeling since a redundancy of LOPs are generated. Chapter IV will address this problem in detail.

2. Azimuth-Azimuth Positioning Method

An azimuth-azimuth positioning method is a direct extension of the standard land surveying practice of azimuth intersection. Ingham [1984, p. 16] states

> This [method] is usually not to be recommended, since complex arrangements are required to ensure that both angles are measured at the instant of the fix marked on the echo-sounder record, and to communicate the fix data to the boat.

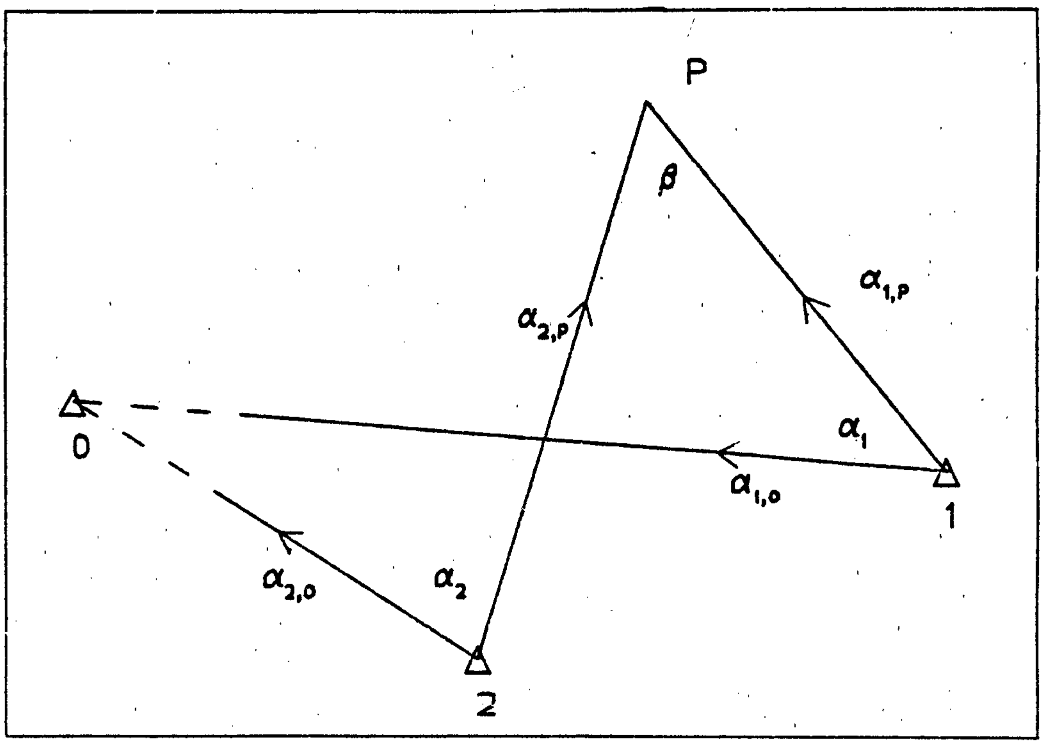

Figure 2.2 Azimuth-Azimuth Positioning Method.

Although true in some instances, good radio communications will alleviate the problem. Another option is to use a flagman aboard the survey vessel to coordinate the activities.

The azimuth-azimuth positioning method uses two theodolites, set up over known control stations, which are sighted upon the survey vessel as it moves along the sounding line. When done correctly, the method produces a very accurate position at the intersection of the visual LOPs. The azimuth to the vessel is obtained by initializing on a known control station and then measuring the clockwise angle to the survey vessel (Figure 2.2). If theodolites occupy the known geodetic shore stations 1 and 2, and station 0 is the station which is initialed upon, then the coordinates of point P, X_P, and Y_P can be determined by the point-slope equations [Davis et al., 1981, p. 371].

$$Y_P = Y_1 + (X_P - X_1) \cot(\alpha_{1,P}) \tag{2.8}$$

and

$$Y_P = Y_2 + (X_P - X_2) \cot(\alpha_{2,P}) \tag{2.9}$$

Subtraction of Equation 2.8 from Equation 2.9 yields

$$X_P = [(Y_1 - Y_2) - X_1 \cot(\alpha_{1,P}) + X_2 \cot(\alpha_{2,P})] / [\cot(\alpha_{2,P}) - \cot(\alpha_{1,P})] \tag{2.10}$$

and

$$Y_P = Y_2 + (X_P - X_2)\cot(\alpha_{2,P}) \tag{2.11}$$

Figure 2.2 shows

$$\alpha_{1,P} = \alpha_{1,0} + \alpha_1 \tag{2.12}$$

and

$$\alpha_{2,P} = \alpha_{2,0} + \alpha_2 \tag{2.13}$$

The use of MLOP obtained from multiple azimuths to determine the position of the vessel is discussed in detail by both Kaplan [1980] and Silva [1982].

3. Hybrid Positioning Methods

A hybrid positioning method (i.e., range-azimuth method) is a frequently used positioning method on near-shore large-scale surveys. The method provides an excellent geometric determination of the vessel's position since the two LOPs always intersect at 90°. One LOP will be generated from a measured electronic range to the vessel. The other LOP is the observed azimuth to the survey vessel. A concentric configuration of the ranging and angle measuring devices is commonly used on large-scale surveys. A concentric configuration is not always possible due to terrain and the location of the control stations. In that case, one of the measurements must be made from an eccentric

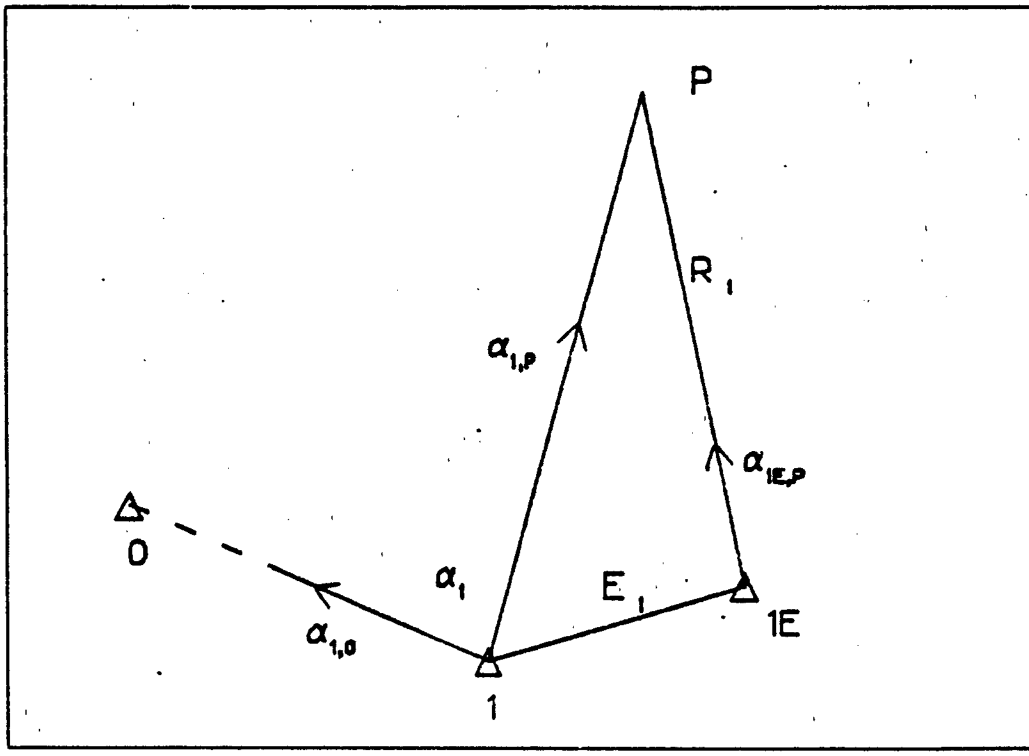

Figure 2.3 Range-Azimuth Positioning Method.

station 1E (Figure 2.3). In practice station 1 and station 1E are within about 2 m of each other and are occupied by a theodolite and a remote ranging unit, respectively.

Knowing the coordinates of stations 1 and 1E, the coordinates of point P may be computed by the equations [Wallace, 1971, pp. 46-48]

$$X_P = X_{1E} + R_1 \sin(\alpha_{1E,P}) \tag{2.14}$$

$$Y_P = Y_{1E} + R_1 \cos(\alpha_{1E,P}) \tag{2.15}$$

where $\alpha_{1E,p}$ is derived from the equation

$$\sin(\alpha_{1E,P} - \alpha_{1,P}) = [1 / R_1]\, [(X_1 - X_{1E}) \cos(\alpha_{1,P}) - (Y_1 - Y_{1E}) \sin(\alpha_{1,P})] \tag{2.16}$$

which reduces to

$$\alpha_{1E,P} = \alpha_{1,P} + \arcsin([(X_1 - X_{1E}) \cos(\alpha_{1,P}) - (Y_1 - Y_{1E}) \sin(\alpha_{1,P})] / R_1) \tag{2.17}$$

B. ERROR THEORY AND ERROR PROPAGATION

The present NOS Hydrographic Manual [Umbach, 1976, p. 1-4] states that positional error on any fix shall seldom exceed 1.5 mm at the scale of the survey. Normally 1.0 mm is allocated for navigational system error and 0.5 mm for plotting error. Thus, for a 1:5,000-scale survey the total allowable navigational error is 5 m. The rated accuracy of most positioning systems used approaches this allowable error. Errors must be identified and dealt with accordingly.

1. Types of Errors

Types of errors as applied to hydrographic surveying are described [Davis et al., 1981; Greenwalt and Schultz, 1962; Heinzen, 1977; Kaplan, 1980; Mikhail, 1976; Perugini, 1984; Waltz, 1983].

a. Blunders

Blunders, also known as gross errors or mistakes, are mainly due to equipment malfunctioning or

observer carelessness. Blunders usually are of large magnitude in comparison to other errors. Unlike land surveying, hydrographic surveying is a dynamic situation and the existence of blunders in field data is commonplace. When using a theodolite for position control of the vessel, the observer rarely has time to check the pointing to the moving target (survey vessel). Also the acquisition of multiple readings and repeating measurements independently to check for consistency, which are standard procedures in land surveys, are not feasible. On any particular fix, an observer has just one chance of an accurate sight to the vessel. If any doubt exists as to the quality of the sighting, the observation should be rejected.

Blunders can occur when using electronic positioning systems due to malfunctioning of the equipment. A problem inherent with short-range systems, such as Del Norte Trisponder, is multipath effects (reflected wave from the ocean surface as well as the direct transmission) [Munson, 1977, p. 4]. Fading, dropout of the ranging signal, and reflected signals are commonly observed during field work due to the omnidirectional antenna aboard the survey vessel.

Detection of blunders is one of the daily functions of a hydrographer. Automated data acquisition systems have the capability of "flagging" blunders on-line

through software engineering. During nonautomated surveying, a plot of the position fixes is maintained to check the accuracy of each fix as data are acquired. Most blunders occur during the nonautomated phase of data acquisition due to the increased amount of hand logging.

Blunders can be minimized by using automatic electronic systems for data acquisition, such as the NOS HYDROPLOT Data Acquisition System (DAS) or the Racal Decca AUTOCARTA II system. Also, all blunders must be detected and resolved prior to the final processing of hydrographic data. Normally, repeated checking of the data will minimize the amount of blunders.

b. Systematic Errors

Systematic errors follow a definite pattern and are generally constant in magnitude and sign throughout a series of observations. The system causing the pattern may be dependent on the instrument, or atmospheric effects [Mikhail, 1976, p. 67]. In hydrographic surveying, systematic errors must be determined and data corrected prior to the final processing adjustment of all the survey records.

Determination of systematic errors in hydrography is accomplished through the calibration of the surveying instruments. Calibration is the process of comparing the observed instrument value with a known

standard. The difference between the observed and known values is the total systematic error present in the system. Once the systematic errors are determined, a "corrector" is applied to the observed values which yields the "true" value of the observations. A corrector is equal in magnitude but opposite in sign to the systematic error.

A systematic error existed within the Del Norte Trisponder microwave ranging equipment used as the positioning system for data acquired for this thesis. The standard field procedure for this equipment was used to compare ranges obtained from the Trisponders to a measured geodetic base line. Differences were recorded and applied during the post processing phase of the survey as correctors to the ranges. Daily calibration checks were made while the vessel was underway to ensure that no major changes had developed, since noticable calibration drifts can occur with time and variation in range [Munson, 1977, p. 4]. Base-line calibrations of short-range systems are normally performed before, during, and after the completion of data acquisition on a hydrographic survey.

Numerous systematic errors arise when using theodolites for range-azimuth and azimuth-azimuth positioning on large-scale surveys. Proper adjustment and alignment of the instrument will eliminate a number of errors. Mikhail [1976, p. 71] lists some of the common

sources of systematic errors associated with the use of theodolites. Initial pointing and eccentricity are two additional sources of errors introduced in hydrographic surveys.

An initial pointing must be made to a known geodetic station since the observer is actually measuring a direction to the survey vessel. Normal practice in the field is to adjust the plate setting to a number other than 000°00'00", usually about 000°00'10". This value must be recorded in the record volume of the survey and applied as a corrector during processing. Should this initial corrector be ignored, or applied in the wrong fashion, a systematic error will be present in all of the observations.

Eccentricity frequently occurs when using a range-azimuth positioning method as it is very difficult to erect a remote ranging unit (i.e., a Trisponder) and a theodolite directly over a horizontal control station. Unless two different height tripods are erected over the station, or a platform built to hold both instruments, an eccentric horizontal station must be established to locate one of the survey instruments. If this offset position is not accounted for, then a systematic error (varying in magnitude with the range and direction to the vessel and the eccentricity of the station) will result. During the data acquisition phase for this thesis, the ranging unit used for

observing the range to the vessel was erected over an eccentric station at three of the four horizontal control stations.

c. Random Errors

Random errors account for the remaining variation in measurements after all blunders have been discovered and removed, and the observations corrected for the known systematic errors. Random errors cannot be modeled mathematically but must be modeled under the laws of probability. The random error is a random variable which can take on a number of possible values depending on the probability involved.

> They result from accidental and unknown combinations of causes beyond the control of the observer. Random errors are characterized by: (1) variation in sign -- positive and negative errors occuring with equal frequency, (2) small errors occurring more frequently than large errors, and (3) extremely large errors rarely occuring [Greenwalt and Schultz, 1962, p. 2].

Statistical treatment of random errors relating to hydrographic applications has been given by Waltz [1983] and Perugini [1984].

2. Accuracy of Large-Scale Survey Methods

Two LOPs from completely automated electronic systems can rarely meet the required accuracy for a 1:5,000-scale survey due to the standard error associated with each LOP [Perugini, 1984]. Common survey practice has

been to use hybrid methods, such as range-azimuth, to reduce this error by introducing an accurate direction measured with a theodolite which will have a 90° intersection with the range LOP. Also completely visual methods can be employed, such as azimuth-azimuth or three-point sextant fixes, for the position determination of the survey vessel. All of the methods employed contain unique standard errors associated with the equipment utilized. This section discusses the rated accuracy of the equipment used for data acquisition for this thesis.

a. Errors in Ranges

The total positional accuracy "shall seldom exceed 1.5 mm at the scale of the survey" [Umbach, 1976, p. 1-4]. Munson [1977, p. 2] has equated "seldom" to be 90% of the time which results in a 1.645-sigma value of 4.5 m on a 1:5,000-scale survey. He further states

> Positioning system accuracies are most commonly stated in terms of error along a line of position. Since the total position error will always be at least $\sqrt{2}$ greater than this, an acceptable positioning system must measure a line of position to at least a 1σ level of 4.5 m / $\sqrt{2}$ = 3.2 m for a 1:5,000 survey.

The Del Norte Trisponder system used for this thesis has a manufacturer stated accuracy of ±1 m. Recent stability testing by NOS on the Model 520 Digital Distance Measuring Unit (DDMU) has shown errors in the order of 3 m over various ranges [Whitsell and Berstis, 1983, p. 4]. The

study also found that the range error was a function of low signal strength which could be simulated by signal attenuation. This is a major problem because

> There is no indication in the Model 520 DDMU display of data output when low signal strength conditions are encountered. Reduced signal attenuation will occur during hydrographic survey operations due to dynamic movement of the master antenna on the survey launch, inclement weather conditions, and the presence of multipath propagation zones. [Whitsell and Berstis, 1983, p. 5]

A 3-m standard error will be used for all range LOPs acquired during the field work for this thesis with a Model 540 DDMU. This unit is similar to the Model 520 DDMU, the only difference being that the 540 has a four-range digital display versus a two-range digital display on the 520. A fixed value over all ranges was chosen rather than one which varies with distance, as is normally defined for precise Electronic Distance Measuring (EDM) equipment in land surveying, to simulate common field practice. As a result, the error of each LOP at a short range (1,000 m) will have a higher bias than a LOP at a longer range (10,000 m) due to the fixed error.

b. Errors in Azimuths

One of the objectives of the work done by Waltz [1983] was to determine the pointing error of the Wild T-2 theodolite when used for range-azimuth hydrographic control. His rigorous treatment of the subject found that an

estimated linear error for the pointing of a T-2 was 1.3 m. Actually the pointing error is a function of the angular resolution of the instrument plus the ability of the observer to track the sounding vessel. Both are a function of the distance to the vessel. For azimuth-azimuth positioning methods, the errors in positioning actually depend upon the distance, the angular resolution, and the angle of intersection of the LOPs [Heinzen, 1977, p. 55].

An important conclusion made by Waltz [1983, p. 81] was

> There exists, for any angle measured with a T-2, a time lag of about one second between angle observations and any measurement made aboard the vessel, including both automatic and manually recorded depth and range data. There is then an associated position error for these measurements, the magnitude of which depends upon vessel speed, which was about two meters for the four knot speed used in this experiment.

With this fact in mind, a standard error associated with azimuth LOPs was computed for the data acquired for this thesis. Hand plots of the position fixes show that the speed of the survey vessel was approximately 5 knots (2.6 m/s). Thus, the 1-second time lag would amount to as much as an additional 2.6 m of error in position due to the vessel velocity relative to the shore station. Consider an example which shows positional error as a function of the range and the angular resolution of the instrument (Figure 2.4). Using the standard conversion formula

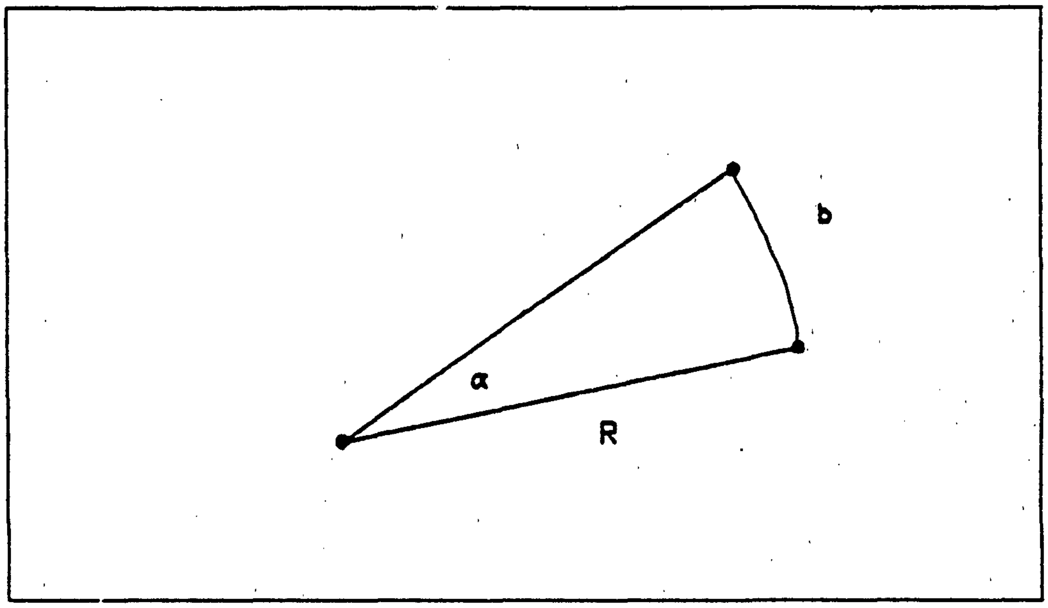

Figure 2.4 Conversion of Angular Measure to Arc Length.

$$\alpha \;/\; \rho \;=\; b \;/\; R \tag{2.18}$$

where b is the arc swept by the vessel (2.6 m), R is the range to the vessel in meters, α is the angular error in seconds, and ρ is a conversion factor (206265"/rad). If R is 1,000 m, then α = (206265")(2.6 m / 1,000 m) = 536" = 8.9'. Likewise, if R is 2,000 m, then α = 4.5'; if R is 3,000 m, then α = 3.0'.

A total standard error for a LOP measured by an azimuth for this thesis was assumed to be 4 m. This value takes into account a 1.3-m pointing error plus the 2.6-m error due to the 1-second time lag at a vessel speed of approximately 5 knots. This was done to duplicate standard

processing techniques for azimuth data By assuming a fixed error over all ranges a reverse situation holds for LOPs from azimuth measurements when compared to the similar situation for range measurements. At longer distances the measured azimuth will be more accurate than at shorter distances because the aspect of the vessel does not change as rapidly. By using a fixed error in position, the azimuths obtained at close range will have an underestimated error whereas those observed at longer ranges will have an error which is overestimated.

3. Error Propagation

Since random errors will exist in the field measurements, then the errors in the computed positions of the survey vessel can be obtained by the law of propagation of errors. Error propagation is the evaluation of errors in the computed positions as functions of the errors in the field measurements. As stated previously, random errors follow statistical laws and can be dealt with through probability distributions of these errors.

Let the function Y be defined by

$$Y = f(X_1, X_2, X_3, \ldots, X_n) \tag{2.19}$$

in which X_1, X_2, X_3, . . ., X_n are uncorrelated measurements. Thus, the law of propagation of errors [Mikhail and Gracie, 1981, pp. 148-178] can be shown as

$$\sigma_Y^2 = (\ \partial Y\ /\ \partial X_1\ )^2\ \sigma_{X_1}^{\ 2} + (\ \partial Y\ /\ \partial X_2\ )^2\ \sigma_{X_2}^{\ 2} + \ .\ .\ .\ + (\ \partial Y\ /\ \partial X_n\ )^2\ \sigma_{X_n}^{\ 2} \qquad (2.20)$$

Consider the problem of defining the position of a survey vessel at sea. The goal of the positioning methods previously discussed is to accurately determine the computed coordinates (X_P, Y_P) of the survey vessel. Each of the two coordinates are independently a function of the measurements used to compute the coordinates. For example, when using a range-range positioning system the coordinates are a function of the measured ranges (R_1 and R_2) and the known station coordinates on shore such that,

$$X_P = f_1(R_1,R_2,X_1,X_2) \qquad (2.21)$$

$$Y_P = f_2(R_1,R_2,Y_1,Y_2) \qquad (2.22)$$

where X_1, Y_1 and X_2, Y_2 are the known coordinates of the shore stations 1 and 2, respectively. The law of propagation of errors takes the following form

$$\sigma_X^2 = (\ \partial X_P/\partial R_1\ )^2\ \sigma_{R_1}^{\ 2} + (\ \partial X_P/\partial R_2\ )^2\ \sigma_{R_2}^{\ 2} + (\ \partial X_P/\partial X_1\ )^2\ \sigma_{X_1}^{\ 2} + (\ \partial X_P/\partial X_2\ )^2\ \sigma_{X_2}^{\ 2} \qquad (2.23)$$

$$\sigma_Y^2 = (\ \partial Y_P/\partial R_1\ )^2\ \sigma_{R_1}^{\ 2} + (\ \partial Y_P/\partial R_2\ )^2\ \sigma_{R_2}^{\ 2} + (\ \partial Y_P/\partial Y_1\ )^2\ \sigma_{Y_1}^{\ 2} + (\ \partial Y_P/\partial Y_2\ )^2\ \sigma_{Y_2}^{\ 2} \qquad (2.24)$$

The computed position (X_P, Y_P) from a range-range positioning system was given by Equations 2.1 through 2.7. Partial differentiation of the terms in Equations 2.23 and 2.24 yields Equations 2.25 through 2.28 where

$$u = (D^2 + R_1^2 - R_2^2) / (2 R_1 D)$$

then

$$\partial X_P / \partial R_1 = \sin(\alpha_{1,P}) - [\cos(\alpha_{1,P})] (1 - u^2)^{-1/2} [(R_1^2 + R_2^2 - D^2) / (2 R_1)] \tag{2.25}$$

$$\partial X_P / \partial R_2 = [\cos(\alpha_{1,P})] (1 - u^2)^{-1/2} (R_2 / D) \tag{2.26}$$

$$\partial Y_P / \partial R_1 = \cos(\alpha_{1,P}) + [\sin(\alpha_{1,P})] (1 - u^2)^{-1/2} [(R_1^2 + R_2^2 - D^2) / (2 R_1)] \tag{2.27}$$

$$\partial Y_P / \partial R_2 = [\sin(\alpha_{1,P})] (1 - u^2)^{-1/2} (- R_2 / D) \tag{2.28}$$

Since the X and Y coordinates of the shore stations are constant then

$$\partial X_P/\partial X_1 = \partial X_P/\partial X_2 = \partial Y_P/\partial Y_1 = \partial Y_P/\partial Y_2 = 0 \tag{2.29}$$

Previously, it was shown that the standard error of the Del Norte Trisponder system, σ_R, has been determined to be 3 m, regardless of range, so

$$\sigma_{R_1} = \sigma_{R_2} = \sigma_R = 3 \text{ m} \tag{2.30}$$

Then, the standard error of each of the computed coordinates will be

$$\sigma_X = \sigma_R \sqrt{(\partial X_P / \partial R_1)^2 + (\partial X_P / \partial R_2)^2} \tag{2.31}$$

and

$$\sigma_Y = \sigma_R \sqrt{(\partial Y_P / \partial R_1)^2 + (\partial Y_P / \partial R_2)^2} \tag{2.32}$$

For range-azimuth positioning systems, the coordinates of the vessel are a function of the measured range and azimuth on each fix plus the X, Y coordinates of the range and azimuth station

$$X_P = f_1(R_1, \alpha_{1,P}, X_{1E}, X_1) \tag{2.33}$$

and

$$Y_P = f_2(R_1, \alpha_{1,P}, Y_{1E}, Y_1) \tag{2.34}$$

so that the equation of the propagation of errors takes the form

$$\sigma_X^2 = (\partial X_P/\partial R_1)^2 \sigma_{R_1}^2 + (\partial X_P/\partial \alpha_{1,P})^2 \sigma_{\alpha_{1,P}}^2 + (\partial X_P/\partial X_{1E})^2 \sigma_{X_{1E}}^2 + (\partial X_P/\partial X_1)^2 \sigma_{X_1}^2 \tag{2.35}$$

$$\sigma_Y^2 = (\partial Y_P/\partial R_1)^2 \sigma_{R_1}^2 + (\partial Y_P/\partial \alpha_{1,P})^2 \sigma_{\alpha_{1,P}}^2 + (\partial Y_P/\partial Y_{1E})^2 \sigma_{Y_{1E}}^2 + (\partial Y_P/\partial Y_1)^2 \sigma_{Y_1}^2 \tag{2.36}$$

The position coordinates (X_P, Y_P) were given in Equations 2.14 through 2.17. The value of $\alpha_{1E,P}$ (Equation 2.17) is repeated here (Equation 2.37) for continuity.

$$\alpha_{1E,P} = \arc\sin\left([(X_1 - X_{1E}) \cos(\alpha_{1,P}) - (Y_1 - Y_{1E}) \sin(\alpha_{1,P})] / R_1\right) + \alpha_{1,P} \tag{2.37}$$

Partial differentiation of the terms in Equations 2.35 and 2.36 where

$$u = (1/R_1)\,[(X_1 - X_{1E}) \cos(\alpha_{1,P}) - (Y_1 - Y_{1E}) \sin(\alpha_{1,P})]$$

yields

$$\partial X_P / \partial R_1 = \sin(\alpha_{1E,P}) - u\,[\cos(\alpha_{1E,P})]\,(1 - u^2)^{-1/2} \tag{2.38}$$

$$\partial X_P / \partial \alpha_{1,P} = [\cos(\alpha_{1E,P})]\,\{R_1 - (1 - u^2)^{-1/2}\,(X_1 - X_{1E})\,[\sin(\alpha_{1,P})] + (Y_1 - Y_{1E}) \cos(\alpha_{1,P})\} \tag{2.39}$$

$$\partial Y_P / \partial R_1 = \cos(\alpha_{1E,P}) + u\,[\sin(\alpha_{1E,P})]\,(1 - u^2)^{-1/2} \tag{2.40}$$

$$\partial Y_P / \partial \alpha_{1,P} = [\sin(\alpha_{1E,P})]\,\{[(1 - u^2)^{-1/2} - R_1]\,(Y_1 - Y_{1E})\,[\cos(\alpha_{1,P})] + (X_1 - X_{1E}) \sin(\alpha_{1,P})\} \tag{2.41}$$

Since the X and Y coordinates of the range and azimuth shore stations are constants then

$$\partial X_P/\partial X_{1E} = \partial X_P/\partial X_1 = \partial Y_P/\partial Y_{1E} = \partial Y_P/\partial Y_1 = 0 \tag{2.42}$$

Substitution into the propagation of error equations (Equations 2.35 and 2.36) results in

$$\sigma_X^2 = (\partial X_P/\partial R_1)^2 \sigma_{R_1}^2 + (\partial X_P/\partial \alpha_{1,P})^2 \sigma_{\alpha 1,P}^2 \quad (2.43)$$

$$\sigma_Y^2 = (\partial Y_P/\partial R_1)^2 \sigma_{R_1}^2 + (\partial Y_P/\partial \alpha_{1,P})^2 \sigma_{\alpha 1,P}^2 \quad (2.44)$$

and the standard error σ_X and σ_Y of the X and Y coordinates is obtained from Equations 2.43 and 2.44.

The standard error of position, σ_P, was used as a reference to classify the accuracy of the data acquired for this thesis. Rather than classify the accuracy using the standard error ellipse or confidence circles, the σ_P value was chosen due to its ease of computation. The value of the standard position error [Saxena, 1972, p. 15] is defined as the square root of the sum of the squares of the propagation variances of the system. Mathematically

$$\sigma_P = \sqrt{\sigma_X^2 + \sigma_Y^2} \quad (2.45)$$

where σ_X^2 and σ_Y^2 are computed as shown previously for range-range LOPs and range-azimuth LOPs. The standard error of position, σ_s in cm, for azimuth LOPs [Mueller, 1979, p. 62] is

$$\sigma_s = (0.485 / \sin \beta) \sigma_\alpha \sqrt{D_1^2 + D_2^2} \quad (2.46)$$

where β is the angle of intersection of the two LOPs; D_1 is the distance from point 1 to point P in km; D_2 is the distance from point 2 to point P in km; and σ_α is the angular error in seconds of arc.

For the overdetermined case of MLOP, the σ_P value (as derived by Equations 4.28, 4.29, and 4.30) will be

$$\sigma_X = \sigma_o \sqrt{Qxx} \tag{2.47}$$

$$\sigma_Y = \sigma_o \sqrt{Qyy} \tag{2.48}$$

and

$$\sigma_P = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sigma_o \sqrt{Qxx + Qyy} \tag{2.49}$$

where σ_o is the standard error of an observation of unit weight, Qxx and Qyy are the diagonal elements of the variance-covariance matrix.

Thus a tag can be placed on every position fix acquired from whatever system, or combination of systems, used to determine the coordinates of the survey vessel. Using this standard position accuracy, a comparison between fixes determined by different systems and methods can be compared so that the most precise combination of ranges, azimuths, or hybrid methods can be analyzed.

III. DATA ACQUISITION PROCEDURES AND SYSTEM PERFORMANCE

The field work required for this thesis involved conducting a hydrographic survey while using MLOP for sounding line position fixing. Two types of positioning data, ranges and azimuths, were acquired simultaneously from four shore control sites. The data were then analyzed using various combinations of ranges and azimuths to determine a more accurate positioning system.

A. DATA ACQUISITION PROCEDURES

Students in the NPS Hydrographic Sciences curriculum conducted a basic hydrographic survey of southern Monterey Bay as part of a field experience requirement. The survey was accomplished using procedures similar to those used by NOS for nearshore surveying. A chartered 36-foot, twin-engine Uniflite boat was the platform used for the survey. During the week of 26 November 1984 to 30 November 1984, the boat was positioned using four ranges and four azimuths from known third-order horizontal geodetic control stations on shore for each position fix of the vessel. The ranges to the vessel were determined from four Del Norte Trisponders and were automatically recorded by a Racal-Decca AUTOCARTA II data acquisition system. The four azimuths were observed with Wild T-2 theodolites. Data acquisition

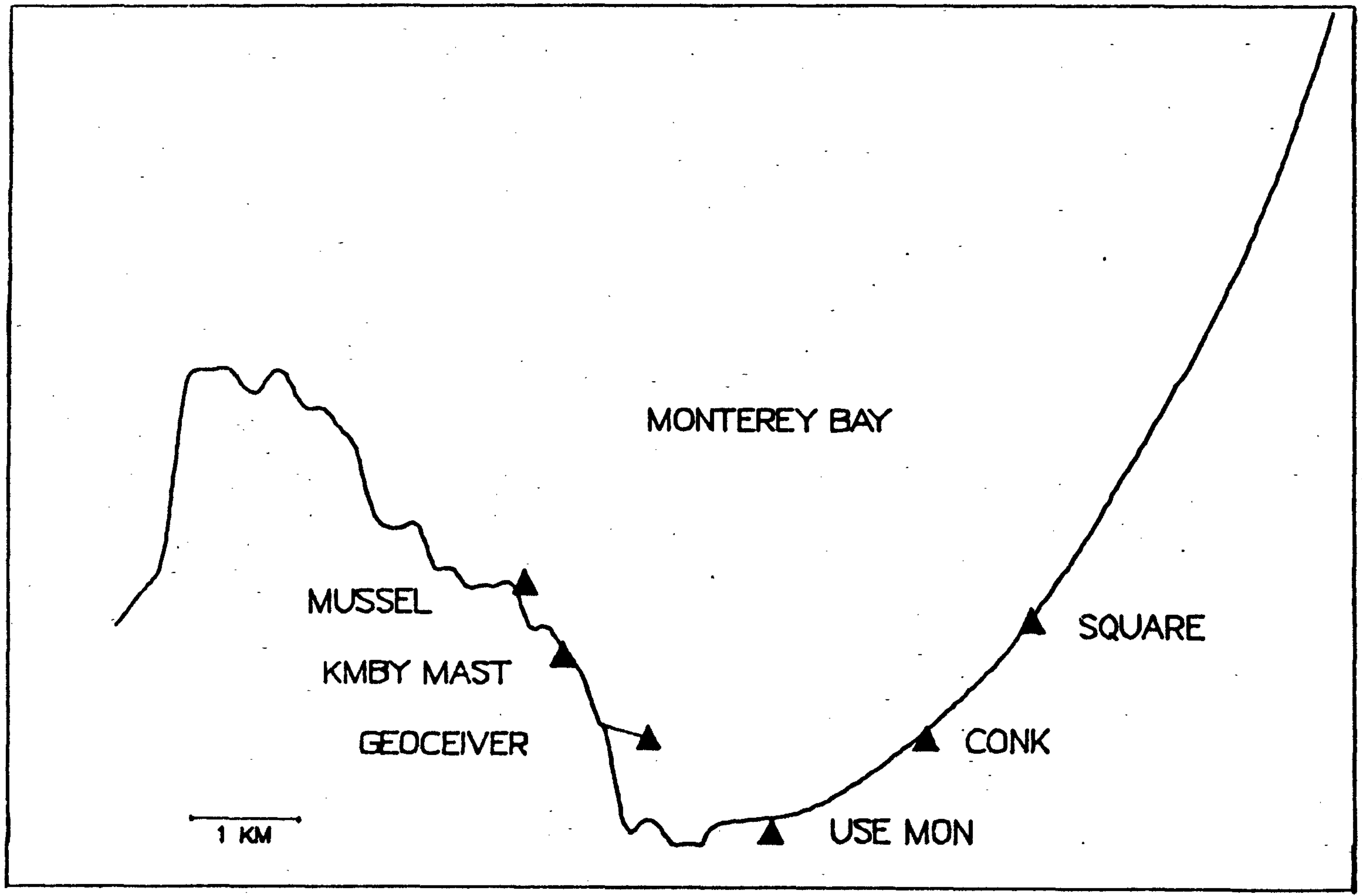

Figure 3.1 Survey Area.

equipment was supplied by Racal-Decca, NOAA, and NPS (Table I).

TABLE I

EQUIPMENT LIST

Aboard M/V Silver Prince

Model 540 DDMU S/N 211 (Racal)
Master Trisponder S/N 3014 (NOAA)

Onshore Control Stations

STATION	REMOTE TRISPONDER	T-2 (S/N)
SQUARE 1984	Code 72 S/N 2819 (Racal)	14405 (NPS)
CONK 1984	Code 74 S/N 2822 (Racal)	51642 (NPS)
USE MON 1978	Code 76 S/N 3004 (NOAA)	30504 (NPS)
GEOCEIVER 1982	Code 78 S/N 2986 (NOAA)	14482 (NPS)

Geodetic control for the survey consisted of third-order monumented stations and established eccentrics (Table II). The eccentric stations were within 2 m of the main station

and marked by masonry nails driven into the concrete. The geodetic position of each eccentric station was computed on a HP 9815 calculator using National Geodetic Survey (NGS) geodetic programs. Initial pointings for the T-2 theodolites were to MUSSEL from stations SQUARE, CONK, and USE MON; and to MONTEREY RADIO STATION KMBY MAST from station GEOCEIVER.

TABLE II

GEODETIC CONTROL POSITIONS

Station	X (m)	Y (m)	Latitude	Longitude
SQUARE	7974.86	3909.43	36°37'07.175"	121°51'00.276"
SQUARE ecc	7976.10	3907.89	36°37'07.140"	121°51'00.230"
CONK	6978.19	2828.77	36°36'32.130"	121°51'40.397"
CONK ecc	6976.70	2827.53	36°36'32.100"	121°51'40.460"
USE MON	5598.98	1982.76	36°36'04.685"	121°52'35.900"
GEOCEIVER	4371.50	2840.28	36°36'32.512"	121°53'25.286"
GEOCVR ecc	4372.99	2839.97	36°36'32.510"	121°53'25.230"
MUSSEL	3220.17	4247.23	36°37'18.151"	121°54'11.628"
KMBY MAST	3641.23	3588.23	36°36'56.789"	121°53'54.678"

The data acquisition procedures were similar to normal survey operations for a large-scale hydrographic survey

using visual control with the addition that on each fix four ranges were recorded automatically by the AUTOCARTA II system. The four theodolite observers trained their instruments on the vessel as it moved along the track line. Position fix marks were communicated over voice radio and the T-2 directions to the vessel were manually recorded by the observers.

To eliminate systematic errors associated with the Del Norte system, the Trisponder units were calibrated over a measured geodetic base line 2565.897 m long (CONK 1984 to REY 1984) on 24 and 25 November 1984. A base-line calibration prior to survey operations is a standard field procedure for this type of equipment. Daily dynamic system checks were made similar to NOS procedures [Holder, 1983]. The observed "drifts" in the Trisponder units were recorded from an ending base-line calibration on 7 December 1984 and were applied as range correctors during the data processing phase of the experiment.

Weather conditions during the 2 days in which horizontal directions to the vessel were recorded was fair. The theodolite observers had difficulty in maintaining an adequate initial pointing due to winds gusting to 30 knots which caused vibration of the tripods. Initial checks were made at the end of each track line and recorded. Observed changes in the initial directions were recorded and applied

during the data processing phase to eliminate any systematic errors in the angles. On the first 2 days of field operations, a total of 176 positions, each with four ranges and four azimuths, were recorded. An additional 175 positions with four ranges were acquired using the AUTOCARTA II system during the last 3 days of the field work.

B. DATA ACQUISITION SYSTEM

The AUTOCARTA II (Figure 3.2) is a portable automated hydrographic data acquisition system. Desired vessel tracks can be preprogrammed into the computer to alleviate the necessity of steering the vessel along range arcs for track control. A real-time graphic copy of the vessel's actual course was plotted from Autocarta II output on a Houston Instruments Model DP-3 Plotter. A Model 540 DDMU recorded the four ranges at 1-minute intervals along the track line. Input and output communication to the AUTOCARTA II were via a modified TI-743KSR keyboard/printer.

C. PERFORMANCE OF DATA ACQUISITION SYSTEM

The operational difficulties experienced during the data acquisition phase of the field work could be described as "standard" for a hydrographic survey of this type. To ensure adequate backup was available for Trisponder units, a total of seven remote and two master units were on hand. Two of the remote units did not responded when interrogated

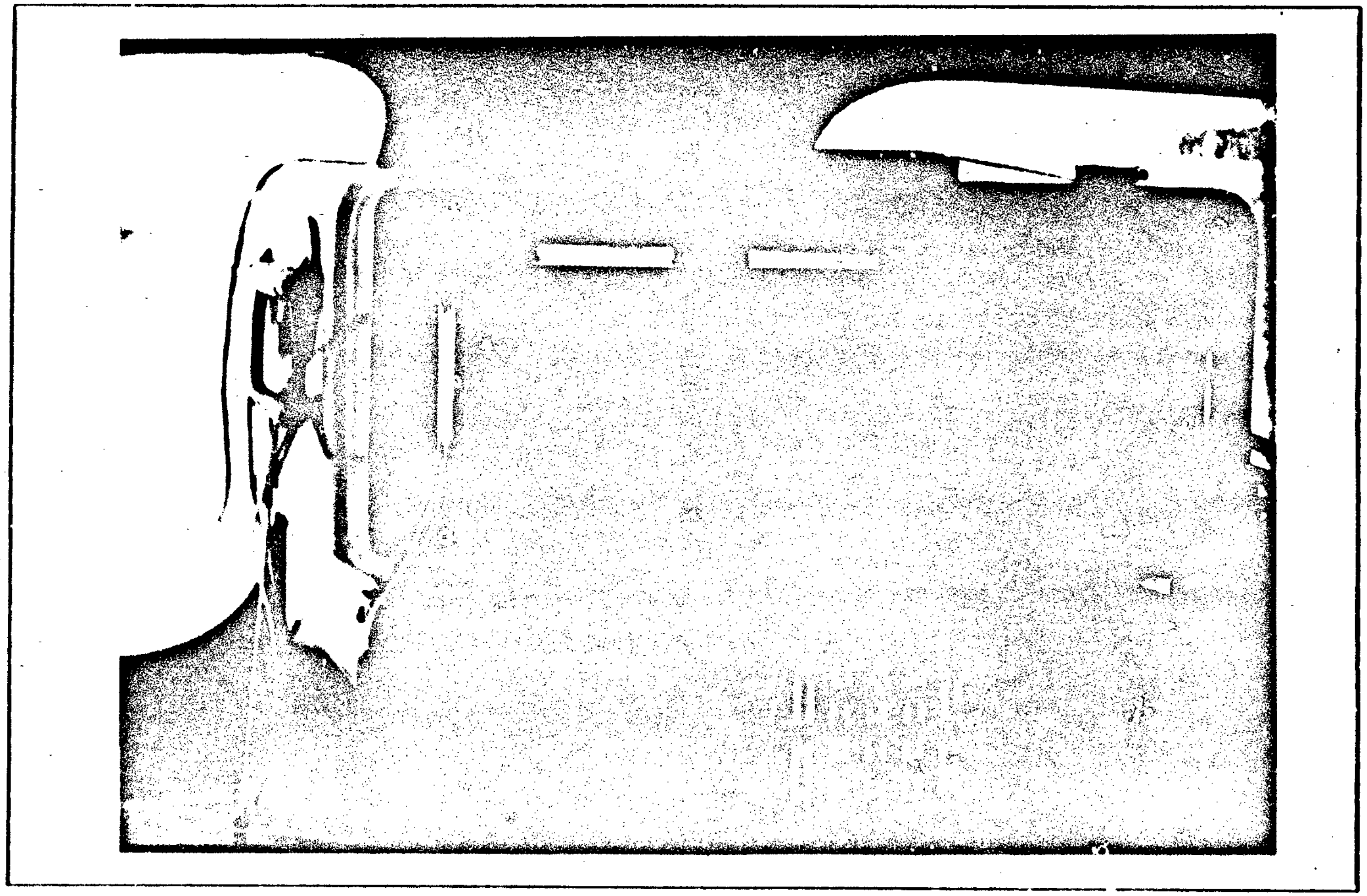

Figure 3.2   Data Acquisition System.

during the base-line calibration of the system and had to be removed for the entire study. New deep-cycle marine batteries were used as power to the 24-volt Trisponder system. These batteries were charged each night to ensure sufficient power during the survey.

A cable linking the AUTOCARTA II to the keyboard/printer was not shipped with the equipment. Therefore, the data were acquired in the normal non-automated fashion by hand recording the range rates as they were displayed on the DDMU on the first day of the survey. The needed cable was available by the next day and the complete automated system was operational for the rest of the survey.

The AUTOCARTA II was set up to interrogate the remote units on shore from left to right when facing shoreward from the vessel which is standard NOS convention. On the first day of data acquisition, day 332, very erratic range rates were being observed on the DDMU from station USE MON 1978 (remote code #76). The update rate was set at 1-second intervals and over half of the time either a totally ambiguous rate or no rate at all was recorded on the DDMU. In normal field operations, a remedy to the situation is to raise or lower the remote unit to eliminate the possibility of "null zones" and "skip zones." The remote unit on station USE MON 1978 was raised and lowered with no change in the performance of the Trisponder observed.

IV. MATHEMATICAL MODEL

A. LEAST SQUARES MODEL

To determine the "best" estimate of a position, redundant observations are necessary so that the method of least squares can be used. The least squares adjustment by observation equations (variation of coordinates) was selected for this thesis (rather than least squares adjustment by condition equations) for reasons of simplicity and clarity [Bomford, 1980, pp. 127-128].

A basic discussion of the method of least squares by observation equations is given below [Bomford, 1980; Mikhail and Gracie, 1981; Cross, 1981; Heinzen, 1977; Kaplan, 1980; Saxena, 1972; Silva, 1982].

1. Unweighted Equations

The equation derivations will be initially accomplished for measurements of equal accuracy.

Let L_i represent independent observed quantities; v_i be residuals of the observed quantities; and x, y, and z be the unknown coordinates of a point in a rectangular coordinate system. Then for each observation, i, an observation equation is formed such that

$$L_i + v_i = f_i(x,y,z) \qquad (4.1)$$

where f_i is a linear or non-linear function. When f_i is a non-linear function, a Taylor series is formed about the approximate values of x_o, y_o, and z_o such that the second and higher order terms are small enough to be neglected. The observation equations then become

$$L_i + v_i = f_i(x_o+dx,\ y_o+dy,\ z_o+dz,\ .\ .\ .) \qquad (4.2)$$

or

$$L_i + v_i = f_i(x_o+dx,\ y_o+dy,\ z_o+dz) + a_i dx + b_i dy + c_i dz + \ .\ .\ . \qquad (4.3)$$

If $\ell_i = L_i - f_i(x_o,y_o,z_o)$, which is the difference between the observed quantities and the computed approximate values of the unknowns, then

$$v_i = a_i dx + b_i dy + c_i dz + \ .\ .\ .\ - \ell_i \qquad (4.4)$$

where

$$x = x_o + dx \qquad (4.5)$$

$$y = y_o + dy \qquad (4.6)$$

$$z = z_o + dz \qquad (4.7)$$

and

$$a_i = \partial f/\partial x,\ b_i = \partial f/\partial y,\ c_i = \partial f/\partial z \qquad (4.8)$$

so that in matrix form the observation equation 4.4 becomes

$$V = AX - L \tag{4.9}$$

which ultimately yields the normal equations

$$NX - U = 0 \tag{4.10}$$

where

$$N = A^T A \tag{4.11}$$

and

$$U = A^T L \tag{4.12}$$

Then substituting into Equation 4.10 and rearranging terms gives

$$X = N^{-1}U = (A^T A)^{-1} A^T L \tag{4.13}$$

which is valid for observations of equal weight.

In actual surveying, computed observations differ from actual observations by a value known as a residual, v. The residual is the difference between an observed value, x_i, and the estimate of the true value, $\hat{x}$, so that

$$\hat{x} = x_i + v_i \tag{4.14}$$

The basic condition of the least squares method is that the sum of the squares of the residuals equal a minimum. Therefore,

$$\Sigma (v_i)^2 = \text{minimum} \tag{4.15}$$

where

$$\Sigma (v_i)^2 = v_1^2 + v_2^2 + . . . + v_n^2$$

for values of equal weight.

2. Weighted Equations

When observations are made utilizing various methods and systems, weights must be applied to account for the lower standard error, σ, and lower variance, σ^2, of the more precise observations. The relationship between weights and σ^2 is

$$w = k / \sigma^2 \tag{4.16}$$

where k is a constant of proportionality having the same value for all observations in one system when the different standard errors of the observations are uncorrelated and of the same precision, as is the case for range-range, azimuth-azimuth, and range-azimuth positioning systems. Thus

$$k = 1 \tag{4.17}$$

Then

$$w_i = 1 / \sigma_i^2 \tag{4.18}$$

which in matrix form is the diagonal matrix

$$W = \begin{bmatrix} 1/\sigma_1^2 & & & & \\ & 1/\sigma_2^2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 1/\sigma_n^2 \end{bmatrix} \tag{4.19}$$

For weighted observations, Equation 4.15 becomes

$$\Sigma\, w_i(v_i^2) = \text{minimum} \tag{4.20}$$

where

$$\Sigma\, w_i(v_i^2) = w_1v_1^2 + w_2v_2^2 + \ldots + w_iv_i^2$$

or in matrix form

$$V^TWV = \text{minimum} \tag{4.21}$$

which after substitution and differentiation with respect to X becomes

$$X = (A^TWA)^{-1}\, A^TWL \tag{4.22}$$

This system of equations is then solved for the matrix X elements which are then added to a provisional set of coordinates at a point, yielding a new pair of coordinates. This process is iterated until the values of the X elements become small enough to meet a specific

tolerance. At this point the method is said to have converged. What remains is the "best" estimate of x and v, designated as $\hat{x}$ and $\hat{v}$.

B. PRECISION OF ADJUSTED VALUES

The first right-hand term of Equation 4.22 is the variance-covariance matrix,

$$(A^TWA)^{-1} = N^{-1} = Q \tag{4.23}$$

and it can be shown that,

$$Q = (A^TWA)^{-1} = \begin{bmatrix} q_{11} & q_{12} & \cdot & \cdot & \cdot \\ q_{21} & q_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & q_{nn} \end{bmatrix} \tag{4.24}$$

The variance-covariance matrix of the least squares estimate of x, $\sigma(\hat{x})$, is

$$\sigma(\hat{x}) = \sigma_o^2\ (A^TWA)^{-1} = \sigma_o^2\ N^{-1} = \sigma_o^2\ Q \tag{4.25}$$

$$\sigma(\hat{x}) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \cdot & \cdot \\ \sigma_{yx} & \sigma_y^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \sigma_n^2 \end{bmatrix}$$

Here σ_o^2 is known as the variance of an observation of unit weight represented as

$$\sigma_o^2 = V^T WV / (n - m) \tag{4.26}$$

The standard error of an observation of unit weight, σ_o, is therefore

$$\sigma_o = \sqrt{V^T WV / (n - m)} \tag{4.27}$$

where n is the number of observations, and m is the number of unknowns. The term (n - m) is known as the degree of freedom.

Consequently, the standard error of the adjusted coordinates, σ_x and σ_y, are given by;

$$\sigma_X = \sigma_o \sqrt{q_{xx}} \tag{4.28}$$

$$\sigma_Y = \sigma_o \sqrt{q_{yy}} \tag{4.29}$$

and the standard position error is

$$\sigma_P = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sigma_o \sqrt{Q_{xx} + Q_{yy}} \tag{4.30}$$

Finally, the semi-major (σ_a), semi-minor (σ_b), and orientation (θ) axes of the error ellipse are given as

$$\sigma_a = \sigma_o \sqrt{Q_{max}} \tag{4.31}$$

$$\sigma_b = \sigma_o \sqrt{Q_{min}} \tag{4.32}$$

where

$$Q_{max,min} = (1/2)\ (Q_{xx} + Q_{yy}) \pm (1/2) \sqrt{(Qxx-Qyy)^2 + 4\ Qxy^2} \tag{4.33}$$

and

$$\theta = (1/2) \text{ arc tan } [\ 2\ Q_{xy}\ /\ (Q_{xx} - Q_{yy})\ ] \tag{4.34}$$

V. DATA ANALYSIS AND RESULTS

A. DATA PROCESSING PROCEDURES

Data acquired in the field were manually logged into data files on the IBM 370/3033AP mainframe computer at NPS via the VM terminals. These data were processed using computer programs (Appendix A) written to be compatible with the VS FORTRAN compiler and used many of the existing library subroutines. The main program which computes the least squares position from three- or four-range LOPs, SLEAST (Appendix B), was composed by Lt. Nick Perugini, NOAA, and the author. The article by Cross [1981] covers the subject in detail. The least squares programs for the azimuth and hybrid MLOP methods are FORTRAN programs LSQAZ4, R3+AZ, R2+AZ2, and RR+AZ (Appendix A). These programs are modifications of programs SILVA1 and SILVA3, written by Silva [1982]. Computation of coordinates and accuracies from two LOPs were made by modifying the program UCOMPS (Appendix A) into RR2LOP, AZ2LOP, RAZ2LOP.

Processing procedures commenced by correcting the observed ranges and azimuths for previously determined systematic errors prior to logging the data into manageable files. The least squares computer programs output the position of the vessel (in X and Y coordinates based on a

Modified Transverse Mercator (MTM) projection[1]); the final residuals; and the computed accuracy, σ_p, of each position fix. The output from the processing programs for two LOPs were the position coordinates of the vessel, the angle of intersection of the LOPs, β, and the standard error of position, σ_p, as determined in Chapter II for each position method.

The geodetic positions of the shore stations used for hydrographic control were converted to MTM coordinates via the utility program UCOMPS (Appendix A).

B. DATA ANALYSIS

The main objective of the thesis was to investigate whether positions computed from MLOP were more accurate than positions determined by only two LOPs. Models and algorithms were developed so field data could be appropriately analyzed. For each positional fix, eight LOPs (four ranges and four azimuths) were measured simultaneously. The eight LOPs were processed separately into ten different sets of positioning methods. Depending

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[1]The MTM projection is a Universal Transverse Mercator projection except the Central Meridian is near the survey area instead of being at a preselected meridian [Wallace, 1971, pp. 3-4]. A Central Meridian of longitude 121°53'00"W was chosen for this thesis. The X coordinate assigned to the Central Meridian, the FEST, was chosen as 5,000 m. The Controlling Latitude, CLAT, or the distance in meters from the equator to some reference latitude, was computed to be 4,050,000 m.

on the method, some of the ten sets were separated into various subsets by using different combinations of observations from the four shore stations.

Let the four shore stations from which the LOPs originated be designated A, B, C, and D. If four-range (or four-azimuth) LOPs are to be used for positioning, there is only one unique combination of four-range (or four-azimuth) LOPs which can solve the problem. Using only three LOPs, there exists four possible combinations of range (or azimuth) LOPs--ABC, ABD, ACD, and BCD.

The problem becomes more confusing with hybrid combinations of range LOPs and azimuth LOPs. A practical limitation was imposed by deciding that each azimuth LOP must originate from the same station (or eccentric station) as the range LOP. Thus, if three LOPs were to be determined from two ranges and one azimuth, the azimuth observation must be made at one of the stations (or its eccentric station) where the ranging unit is located. For example, if Trisponders are set on stations A and B (or the eccentric stations AE and BE). The azimuth LOP must originate from either station A or station B. There are a total of 12 LOP combinations which must be considered when using three ranges and one azimuth, or two ranges and one azimuth (Table III).

TABLE III

LOP COMBINATIONS FOR TWO HYBRID METHODS

3 Ranges & 1 Azimuth		2 Ranges & 1 Azimuth	
3 Ranges	Azimuth	2 Ranges	Azimuth
ABC	A	AB	A
ABC	B	AB	B
ABC	C	AC	A
ABD	A	AC	C
ABD	B	AD	A
ABD	D	AD	D
ACD	A	BC	B
ACD	C	BC	C
ACD	D	BD	B
BCD	B	BD	D
BCD	C	CD	C
BCD	D	CD	D

All possible combinations of the 10 data sets were processed resulting in a total of 56 positions being computed for most of the fixes (Table IV).

Gaps exist between consecutive fix numbers in the output files (Appendices B, C, and F) due to spurious ranges or unresolved blunders which were rejected. Data were rejected if the standard error of position exceeded 3σ for that LOP.

TABLE IV

DATA FILE COMBINATIONS

Method	Possible Combinations
<u>4 LOPs</u>	
4 R	1
4 Az	1
3 R & 1 Az	12
2 R & 2 Az	6
<u>3 LOPs</u>	
3 R	4
3 Az	4
2 R & 1 Az	12
<u>2 LOPs</u>	
2 R	6
2 Az	6
1 R & 1 Az	4
TOTAL	56

The following assumptions were made concerning the overall accuracy determination of each position:

(a) All LOPs were independant and uncorrelated.

(b) All data were free of blunders and systematic errors.

(c) The standard error of range LOPs was 3 m at all ranges.

(d) The standard error of azimuth LOPs was 4 m.

(e) The measured ranges were referred to the same point of time during interrogation by the DDMU time-deskew function [Anonymous, 1982].

(f) Measurements by the azimuth observers were all of equal weight.

1. Range Measurement Methods

An analysis was made of the acquired range data by the number of LOPs. The mean value of σ_p from each positioning method is illustrated in Table V.

a. MLOP

FORTRAN program SLEAST (Appendix B) processed the three- and four-range LOPs (Appendix C) using the least squares method. The mean σ_p value for each day of data and the minimum and maximum σ_p were computed (Table V). For the 3 days in which range measurements were acquired, an average σ_p of 1.3 m was attained with the four LOPs and 1.5 m with the three LOPs. The geometric accuracy of the MLOP fixes exceeded the standard error of the Del Norte system by at least 1.5 m. The slight variation in the mean σ_p's for the three LOPs was due to the geometrical variation of the ranging net from each shore station.

b. Two LOPs

The two-range LOPs were processed by program RR2LOP (Appendix A) to compute the coordinates of the vessel plus the standard error of position, σ_p, for each fix. Analysis of the data shows a direct relationship between the

TABLE V

POSITIONAL ACCURACIES FOR RANGE LOPS

LOPs	DAY	σ_P MIN (m)	σ_P MAX (m)	σ_P MEAN (m)	ANGLE OF INTERSECTION (2 LOPs)
4 R	332	0.3	2.8	1.2	
4 R	333	0.4	2.9	1.4	
4 R	334	0.5	2.4	1.3	
3 R	332	0.1	3.6	1.1	*
		0.04	4.5	1.7	*
3 R	333	0.04	5.7	1.3	*
		0.01	3.7	2.0	*
3 R	334	0.1	2.4	0.8	*
		0.4	4.5	2.2	*
2 R	332	4.8	7.9	6.0	147°
		7.7	236.7	29.2	2°
2 R	333	4.4	8.1	5.7	28°
		4.4	578.2	12.3	1°

* NOTE: For positions which could be computed by more than one combination of LOPs (e.g., 4 combinations of 3-range LOP fixes exist when 4 ranges have been measured), the upper line for each day shows the combination of LOPs giving the minimum mean σ_P's and the lower line for each day shows the combination of LOPs giving the maximum mean σ_P's.

angle of intersection of the LOPs and σ_p, as would be expected. A wide variation in the computed σ_p's exists due to the varying angle of intersections. The pre-selected sounding line orientation was programmed into the AUTOCARTA II prior to the acquisition of data. Since the purpose of the field work was to determine the position of the vessel using MLOP, the angle of intersection of each individual pair of LOPs was ignored. Thus at fix number 19 the computed σ_p was 237 m due to an angle of intersection of 1.6°. During normal survey operations this situation would not be accepted and alternate plans would be made to obtain coverage in that area. Herein lies a benefit of using MLOP since the position of the vessel can be determined from selected shore stations which yield the best geometric configuration. As could be expected, the required accuracy of at least 5 m was not met when using two range LOPs with a standard error of 3 m (Table V).

Overall the capability of acquiring MLOP automatically from measured ranges has proven to be a very viable system. Mean accuracies from 1.2 to 1.6 m were observed depending on the number and geometric combinations utilized. With these accuracies all nearshore positioning requirements can be met.

Using range MLOP for hydrographic positioning has many advantages. The capability of running a fully

automated survey with systems such as the AUTOCARTA II not only increases the productivity but also reduces blunders, thereby increasing the quality of the survey. Once remote transponders are erected over the control stations the need for attendant operators is negated thus reducing the number of personnel required to acquire data. The portability of the system (except possibly for the batteries) is an additional advantage in rugged terrain.

2. Azimuth Measurement Methods

The use of an angle measuring instrument, such as a T-2 theodolite, for positional control of sounding lines is a laborious task. Continuous tracking of the vessel is difficult because the horizontal tangent screw has a finite range. Observers can be tracking the vessel with the tangent screw only to find the end of the drive mechanism seconds before the position fix. Thus the problem of tracking a vessel who's aspect is changing rapidly with respect to a nearby observer is magnified. Older T-2 theodolites have an inverted image of the target which may confuse an observer since the vessel seems to be moving in the opposite direction. Also the reading of the observed direction is through a secondary scope. Therefore, an observer must stop tracking the vessel, align the horizontal plates, read the direction and record the value. The lack of digital telemetering capabilities requires the observed

value to be relayed to the survey vessel by voice radio. Obviously, there exists a potential for many sources of errors if position fixes are required at frequent (30-second) intervals.

Using nonautomated systems (i.e., visual azimuths) restricts position fixing of the vessel to well spaced intervals (minimum of 30 seconds). Intermediate positions along the sounding line must be interpolated via dead-reckoning. In most cases, the actual track of the survey vessel does not exactly correspond to the plotted survey line. On a 1:5,000-scale survey, this additional error could exceed the specified accuracy requirements.

Although the above limitations seem to make the method too complicated and error prone for hydrographic control, it is still one of the most exact positioning methods available for large-scale surveys. Experienced observers have little difficulty in acquiring accurate data.

a. MLOP

Even though a 4-m standard error was applied to all azimuth LOPs, the least squares position from three or four azimuths proved to be the most accurate method investigated. Examination of the tabulated σ_p's (Table VI) shows a mean value of 0.7 m on day 332 and 1.0 m on day 333 when the four LOPs were used. The mean σ_p's from three LOPs ranged from 0.7 m to 1.3 m over all combinations from

TABLE VI

POSITIONAL ACCURACIES FOR AZIMUTH LOPS

LOPs	DAY	σ_P MIN (m)	σ_P MAX (m)	σ_P MEAN (m)	ANGLE OF INTERSECTION (2 LOPs)
4 Az	332	0.3	1.7	0.7	
4 Az	333	0.1	2.9	1.0	
3 Az	332	0.00	2.5	0.7	*
		0.01	9.5	1.2	*
3 Az	333	0.00	4.9	0.8	*
		0.05	4.3	1.3	*
2 Az	332	5.7	8.9	6.3	41°
		6.3	423.6	26.7	1°
2 Az	333	5.7	12.9	6.4	26°
		5.7	84.2	8.6	4°

* NOTE: For positions which could be computed by more than one combination of LOPs (e.g., 4 combinations of 3-azimuth LOP fixes exist when 4 azimuths have been observed), the upper line for each day shows the combination of LOPs giving the minimum mean σ_P's and the lower line for each day shows the combination of LOPs giving the maximum mean σ_P's.

the 2 days of data. The accuracy capabilities of the method are evident.

b. Two LOPs

Although the least squares adjusted positions of the azimuth MLOP proved to be extremely accurate, the two-azimuth-LOPs condition suffers from the same degradation in accuracy as the two-range-LOPs case. The error in position, σ_s in cm, was determined from Equation 2.46. Since the standard error of an azimuth LOP has been defined as 4 m throughout this thesis, σ_α must be evaluated. From the standard conversion formula (Equation 2.18) then

$$\sigma_\alpha = (b / R) \, 206265 \text{ s/rad} \qquad (5.1)$$

where R is the distance in m, and b is the standard error of 4 m. Since the azimuth LOPs result in Xp and Yp, the coordinates of the vessel, then the distance from the computed coordinate to each shore control station is

$$d1 = \sqrt{(X_1 - Xp)^2 + (Y_1 - Yp)^2} \qquad (5.2)$$

and

$$d2 = \sqrt{(X_2 - Xp)^2 + (Y_2 - Yp)^2} \qquad (5.3)$$

Rather than compute separate error values for each distance an average of the distance from both stations, d_a, was used where

$$d_a = (d1 + d2) / 2 \qquad (5.4)$$

and Equation 5.1 becomes

$$\sigma_\alpha = (4 / d_a) 206265 \qquad (5.5)$$

which can be substituted into Equation 2.46 to compute σ_s (in cm). Consequently, the computed σ_p (σ_s in cm times 100) is a function of both the range to the vessel and the angle of intersection of the two LOPs.

Analysis of all of the σ_p's attained in this fashion shows a mean value ot about 6.3 m for all data acquired (Table VI). Although this is rather high for an error in position from two accurate azimuths, the cause can be easily explained. The initial assumption that each azimuth LOP was in error by 4 m at all ranges is rather excessive for theodolite measurements. A more realistic investigation of the angular error of dynamic azimuth measurements should be considered in the future. Standard field procedure for azimuth hydrographic control is to ignore the stated random error of the theodolite because the measured directions are only read to the nearest minute of arc. The only errors associated with a position determined by azimuths are the initial correctors and any blunders discovered during data acquisition and post processing.

3. Hybrid Methods

Numerous combinations of control configurations exist when ranges and azimuths are mixed together. The data acquired were processed by FORTRAN programs R3+AZ, R2+AZ2, and RR+AZ (Appendix A).

a. Three-Range and One-Azimuth LOPs

Three-range and one-azimuth LOPs exhibit larger mean values of σ_p's than in the prior MLOP methods (Table VII). One explanation for this difference is the geometry of the fix has a greater effect on the hybrid methods than on the other methods of position control. In this situation, one intersection is at 90° and up to four intersections are at arbitrary angles depending on the position of the vessel relative to the shore stations. If the angle of intersection of the range LOPs is too large (greater than 150°) or too small (less than 30°) then the quality of the fix may be degraded. The 11.8-m and 12.5-m maximum values for the σ_p's observed on both days (days 332 and 333, Table VII) were found using configuration ABDD (three ranges from stations A, B, and D and the azimuth from station D). In both instances, the angle of intersection between the LOPs from stations A and B was greater than 150°. Therefore, the hybrid method is very sensitive to the angle of intersection of each LOP pair where a range and an azimuth are not taken from the same station.

b. Two-Range and Two-Azimuth LOPs

Analysis of the results obtained from processing the two-range and two-azimuth LOPs found not as much variability in the range of the σ_p's as in the σ_p's of three-range and one-azimuth LOPs. An overall mean σ_p of

TABLE VII

POSITIONAL ACCURACIES FOR HYBRID LOPS

LOPs	DAY	σ_P MIN (m)	σ_P MAX (m)	σ_P MEAN (m)	ANGLE OF INTERSECTION (2 LOPs)
3 R & 1 Az	332	0.1	11.8	1.1	*
		1.2	6.3	3.0	*
3 R & 1 Az	333	0.1	6.7	1.7	*
		1.2	12.5	5.5	*
2 R & 2 Az	332	0.3	2.2	0.8	*
		1.2	5.3	2.1	*
2 R & 2 Az	333	0.2	3.5	1.4	*
		0.2	5.0	2.4	*
2 R & 1 Az	332	0.01	5.1	0.8	*
		0.3	4.9	1.9	*
2 R & 1 Az	333	0.02	4.2	1.1	*
		0.1	7.7	2.6	*
1 R & 1 Az	332	5.0	5.0	5.0	90°
1 R & 1 Az	333	5.0	5.0	5.0	90°

* NOTE: For positions which could be computed by more than one combination of LOPs (e.g., 4 combinations of 3-range LOP fixes exist when 4 ranges have been measured), the upper line for each day shows the combination of LOPs giving the minimum mean σ_P's and the lower line for each day shows the combination of LOPs giving the maximum mean σ_P's.

1.5 m on day 332 and 1.9 m on day 333 was observed. Observing both a range and an azimuth, which intersect at 90°, at each of the two stations causes the σ_p values to be more consistent. When the two normal LOPs from one station are combined with two normal LOPs from another station a strong position is determined. Since an angle of intersection of 90° is the best possible geometry for a two-LOP fix, then this position is made from two accurate sets of LOPs.

c. Two-Range and One-Azimuth LOPs

Overall mean values of σ_p of 1.4 to 2.0 m were determined when two-range and one-azimuth LOPs were used for positioning. Variability exists in the range of σ_p values observed due to the geometry problems stated previously. Two of the LOPs intersect at 90° and then a third LOP is added to this accurate configuration to determine the three-LOPs fix.

d. One-Range and One-Azimuth LOPs

The observed σ_p for the two-LOP Range and Azimuth method varied very little due to the minute change in the intersection angle of the LOPs. Since the ranges were measured from eccentric stations for three of the four control sites, the angle of intersection was slightly different than 90°. A value of 5 m for the standard error of position was determined for most of the position fixes.

Due to the consistency of the intersection angle of the LOPs, the range-azimuth LOPs method is prefered for large-scale hydrography. With the present interpretation of the IHO specifications, the 5-m σ_p would be acceptable for positional accuracy.

C. COST COMPARISONS OF POSITIONING METHODS

The increased cost of using MLOP may outweigh the benefits of an increase in accuracy. Conventional two-LOP methods require the minimum equipment and personnel to produce a hydrographic survey. The continuing increase in marine positioning requirements may force the hydrographer of the future to decide what price should be paid to produce a more accurate survey.

Component costs of the equipment used in this thesis were obtained in June 1985 from manufacturer representatives (Table VIII). Prices quoted are the government contract rate in U.S. dollars. Table IX combines the costs of all the components depending on the method used to position the vessel. Only the four-LOP and two-LOP methods were addressed as the costs of the other methods can be inferred. The costs for the two-LOP method assumes the data will be acquired by the conventional nonautomated procedure commonly used on large-scale surveys. Equipment which is common to all methods (i.e., vessels, tripods, signals, etc.) have been ignored in the tabulation.

TABLE VIII

COMPONENT EQUIPMENT COSTS

COMPONENT	COST($)
Del Norte Trisponder:	
Model 542 DDMU	14,500
Master	9,500
Remote	9,600
RACAL-DECCA AUTOCARTA II	24,750
Houston Instrument Plotter	6,500
Wild T-2 Theodolite	6,500
Portable VHF Radio	800

The price of equipment for the four-range MLOP automated method approaches $100,000. In comparison, equipment for the four-azimuth MLOP method is about one-third the price of the four-range MLOP method. The difference in equipment costs in the two methods is very deceptive. To acquire four azimuths, four observers are needed to track the vessel. The cost for this additional personnel over the automated ranging system could approach the difference in equipment costs in just over one year. The labor costs involved would

TABLE IX

COST COMPARISON OF EQUIPMENT

Four-Range MLOP	
540 DDMU	$14,500
Master	9,500
Remotes (4 @ $9,600)	38,400
AUTOCARTA II	24,750
Plotter	6,500
TOTAL	$93,650
Four-Azimuth MLOP	
T-2 Theodolites (4 @ $6,500)	26,000
Portable Radios (4 @ $800)	3,200
TOTAL	$29,200
Two-Range & Two-Azimuth MLOP	
540 DDMU	14,500
Master	9,500
Remotes (2 @ $9,600)	19,200
AUTOCARTA II	24,750
T-2 Theodolites (2 @ $6,500)	13,000
Portable Radios (2 @ $800)	1,600
TOTAL	$82,550
Two-Range LOPs	
540 DDMU	14,500
Master	9,500
Remotes (2 @ $9,600)	19,200
AUTOCARTA II	24,750
TOTAL	$67,950
Two-Azimuth LOPs	
T-2 Theodolites (2 @ $6,500)	13,000
Portable Radios (2 @ $800)	1,600
TOTAL	$14,600
Range and Azimuth LOPs	
540 DDMU	14,500
Master	9,500
Remote	9,600
T-2 Theodolite	6,500
Portable Radio	800
TOTAL	$40,900

depend upon the skill and experience of the personnel used to acquire the data. For four observers paid $8.00 per hour, for an 8-hour day, for 260 days per year, the increase due to labor would be $66,560 (4 x 8 x $8 x 260) per year. Added to the equipment costs of $29,200, the total price exceeds $95,000.

Another expense associated with the four-azimuth MLOP method would be the cost of the hardware needed to process the data using a least squares algorithm. A data processing computer would be necessary to perform the repeated iterations required by the algorithm. Additionally, a major disadvantage of the nonautomated data acquisition method is that observed azimuths must be hand logged into the data processing system. This manual intervention increases the existance of numerous blunders and involves additional labor costs which could approach the cost of acquiring the four-azimuth LOPs.

With these facts in mind, the overall costs of the four-azimuth method could approach the cost of the automated four-range method. Given this circumstance, the optimal system would be to use an automated ranging method to increase production, reduce blunders in recording the observations, and have better control of the survey operations.

D. OPTIMIZED SYSTEM

One of the objectives of this thesis was to determine an optimal system for the acquisition of positional data in hydrography. Results from the data analysis proved conclusively that MLOP do improved fix accuracy over two LOPs. Every effort was made to duplicate the actual field work currently done by NOS so that a valid comparison of methods could be made.

To adequately discuss an optimal system a few considerations must be made. First, a consideration of the accuracy requirements must be made. This accuracy requirement will be dependent upon the purpose of the survey. Even with the initial accuracy estimates used for this thesis, all of the MLOP and the range-azimuth two-LOP methods will meet the 5-m positional accuracy. Therefore, the optimal system would be dependent upon the final product the hydrographer wishes to produce. If automation is the overriding criterion, then a ranging MLOP method would adequately fulfill the needs of the survey. An automated system, such as AUTOCARTA II, can be bought or leased off-the-shelf, to provide the user with an automated MLOP system. Automation frees personnel to do other tasks since the need for shore observers is eliminated (except for station maintenance). By acquiring the data automatically on board the survey vessel, the existance of blunders are

minimized, thereby decreasing the processing time. Since the capability exists for pre-selecting track lines and controlling the movements of the vessel, more production is possible with an automated system. Overall the use of range MLOP and an automated DAS is the most feasible method to conduct accurate near-shore hydrography.

Second, if the purpose of the survey dictated that the accuracy of position must be to within 1 m, or less, then more effort must be made to acquire this accuracy. Even when a 4-m standard error was used for each LOP, this thesis found that the σ_p for 63% of the positions determined by four- and three-azimuth LOPs were less than or equal to 1.0 m. Although the method is susceptible to numerous errors, is labor intensive, and logistically complex, one could produce a survey of a nearshore area to within 1.0-m accuracy with good field procedures.

Third, cost must be considered. The benefit of using just two LOPs is that minimal equipment and labor costs are involved to acquire adequate data. The use of MLOP will increase the initial cost of equipment at least twofold. The cost may be prorated over time so in 4 to 5 years the benefit of producing a higher quality product would compensate the hardware costs. Additionally, the cost to establish more shore control must be considered. More LOPs would mean more control stations are required than presently

needed for conventional hydrographic surveys. Detailed survey planning could minimize the cost by choosing the optimum number of stations needed for the method of data acquisition.

VI. CONCLUSIONS

A. MLOP FOR ON-LINE CALIBRATION OF RANGING SYSTEMS

Short-range microwave survey systems, such as the Del Norte Trisponder, require a static base-line calibration over a known geodetic distance to eliminate systematic errors. Daily system checks must also be made during the course of the survey to detect any calibration drifts over time. Daily system checks can be made in either a static or dynamic mode. Static checks can be made by taking fixed point observations alongside a known point, three-point sextant fixes with a check angle, theodolite intersections from shore control, or range-azimuth positions using a total station EDM unit. Dynamic checks can include steering a range with predetermined crossing angles or underway simultaneous theodolite intersections [Holder, 1983].

Experience has shown that whatever system is used to perform daily calibration checks of the equipment, a substantial amount of time is consumed on this exercise in quality control. Weather permitting, daily system checks are made prior to and after each day's hydrography. Under ideal conditions, the amount of time consumed to obtain the required calibrations usually is 1 hour of the survey day.

A distinct advantage of using MLOP for position control in hydrography is that redundant data are acquired simultaneously on each fix. By having a third or fourth LOP, errors in position could be detected more readily than in the conventional survey mode where everything was assumed to be correct until it was too late. Thus the need to halt survey operations to make a daily system check could be eliminated. Suppose four-range MLOP were to be used to position the vessel during a survey. Once the system was on-line and warmed-up, a number of positions could be obtained by the automatic system while underway to the working grounds. If a large standard error of position, σ_p, was determined by the software within the computer, then an examination of the residuals for each LOP could be examined to determine which range or ranges were in error. This error could be from equipment malfunctions, errors in the locating of shore stations, or erroneous ranges due to low signal strength, reflections, etc. Additional fixes could confirm, or reject the existance of a substantial error. Once isolated, the unit (or units) causing the error can be removed or replaced. Here the advantages of having four-range MLOP as control can be realized. Should one of the units fail, there would still be a redundant range acquired allowing for a least squares computation. If for some reason two units failed, operations could continue using conventional two-range LOPs.

B. FUTURE ASPECTS AND CONSIDERATIONS

The use of MLOP for hydrographic sounding line control should be the primary method for future nearshore surveys. As advances in electronic technology continue, the economy of miniature computer installations will make automated hydrographic data acquisition systems more feasible. NOS is presently in the process of revitalizing the outdated HYDROPLOT/HYDRLOG systems onboard its hydrographic vessels. The Shipboard Data System III (SDS III) is currently being built to increase automation and acquisition of hydrographic data. One of the capabilities of SDS III will be to acquire and process MLOP for sounding line positions.

The methods presented in this thesis for data acquisition are based on the use of components, such as Del Norte Trisponders, which have been in existence since the early 1970's. This thesis has demonstrated that using these systems in a MLOP configuration will substantially increase the accuracy of the survey. However, other systems exist which should be considered in conjunction with conventional MLOP.

Krupp-Atlas Elektronik introduced a new short-range dynamic positioning system, Polarfix, in 1982. The advertised [Wentzell, 1983] advantages of the system are:

(a) Range-Azimuth fixing to 0.1-m accuracy.

(b) Automatic operator-free tracking of the vessel.

(c) Portable shore stations.

Since the Polarfix system has the potential to alleviate some of the accuracy problems associated with the present two-LOP methods, additional studies should be made to ascertain the manufacturer's claim as to the proported accuracy of the system.

Differential GPS is a method which should be considered for large-scale hydrographic surveys. The theory has proven GPS extremely accurate (within centimeters) for control sites ashore where receivers acquire positional data from numerous passes of the satellites. Accuracies on the order of 2 to 3 m may be attained underway once the theory and technology have been developed.

Future positioning systems may be integrated so that a combination of methods and systems can be analyzed to determine the "best" fix.

APPENDIX A

FORTRAN PROGRAMS FOR DATA PROCESSING

The following listed programs were used to process the LOP data for this thesis. Output files were generated to list the standard error of the adjusted coordinates, σ_x and σ_y; the correlation coefficient, σ_{xy}; the semi-major and semi-minor axes of the error ellipse, σ_a and σ_b; the orientation of the error ellipse, θ; and the standard error of position, σ_p.

AZ2LOP

Program AZ2LOP computes the positional accuracy of a fix obtained from two azimuth LOPs. This program is a modificaton of program UCOMPS.

LSQAZ4

Program LSQAZ4 computes positional accuracy using four azimuth LOPs. This program is a modification of program SILVA1 [Sliva, 1982]. The modification permits handling large amounts of data.

RAZ2LOP

Program RAZ2LOP computes positional accuracy of a fix obtained from one range LOP and one azimuth LOP. This program is a modificaton of program UCOMPS. It includes the error propagation algorithm.

RR+AZ

Program RR+AZ computes the positional accuracy of a fix obtained from two range LOPs and one azimuth LOP. This program is a modification of program SILVA3 [Sliva, 1982]. The modification permits handling large amounts of data.

RR2LOP

Program RR2LOP computes positional accuracy using two range LOPs. This program is a modification of program UCOMPS.

R2+AZ2

Program R2+AZ2 computes the positional accuracy of a fix obtained from two range LOPs and two azimuth LOPs. This program is a modification of program SILVA3 [Silva, 1982]. The modification permits acceptance of another azimuth LOP.

R3+AZ

Program R3+AZ computes the positional accuracy of a fix obtained from three range LOPs and one azimuth LOP. This program is a modification of program SILVA3 [Silva, 1982] which computed a least squares position from two range LOPs and one azimuth LOP.

SLEAST

Program SLEAST (Appendix B) computes the least squares position and positional accuracy of a fix from three or four

range LOPs. The program was written by Lt. Nick Perugini and the author from the algorithm presented by Cross [1981].

UCOMPS

Program UCOMPS computes coordinates of a point in either Geographical Position (G.P.) or X and Y grid coordinates from range-range, range-azimuth, or azimuth-azimuth LOPs. It is a utility package program used by Hydrographic Sciences students at NPS.

APPENDIX B

FORTRAN PROGRAM FOR LEAST SQUARES POSITION FROM MULTIPLE RANGES

```
C  PROGRAM SLEAST:  RUN IN WATFIV or FORTRAN
C
C
C  THIS PROGRAM TAKES MULTIPLE LOPS IN THE FORM OF RANGES FROM
C  FOUR SHORE STATIONS AND COMPUTES A LEAST SQUARES POSITION.
C  INPUT THE STATIONS COORDINATES IN THE FIRST FEW LINES OF THE
C  PROGRAM.  THE PROGRAM CAN EASILY BE MODIFIED TO TAKE A
C  NUMBER OF LOPS LESS THAN OR GREATER THAN FOUR. JUST INPUT
C  COORDINATES OF XSTA(I) AND YSTA(I). LIKEWISE WITH RANGE(I)
C  CHANGE NUM=4 TO NUM = NUMBER OF LOPS. PROGRAM WILL TAKE UP TO
C  10.  ONE FEATURE OF THIS PROGRAM IS IT COMPUTES A BEST
C  ESTIMATE OF POSITION FROM ALL THE INTERSECTING LOPS, BASED
C  ON STRONGEST ANGLE OF INTERSECTION.
C
C  LIBRARY SUBROUTINES ARE APPENDED TO PROGRAM.
C
C
      REAL * 8 XSTA(10),YSTA(10),RANGE(10),XOSTA(10),YOSTA(10),
     *RANGEO(10),STICK,X1,X2,Y1,Y2,BB,CC,ANG,DIFF,HOLD1,HOLD2,
     *HOLDIS,R1,R2,AA,XNO1,XNO2,YNO1,YNO2,ADJ,CANG,NANG,PANG,
     *DIFX,DIFY,FINX,FINY,WK(22),C(4),CRAY(10),BRAY( 4),ARAY(4,2)
     *ATA(2,2),ATB(2,1),DEL(2,1),NICK(4 ) ,VRAY(4),LANG,CCC,FF,
     *COR(4),WT(4,4),ATW(2,4),SIGNO,SIGX,SIGY,SIGXY,RHO,VTWV(1,1)
      REAL * 8 SIGMA(10),ATWA(4,4),ATWB(2,1),VTW(1,4),Q(2,2),
     *,SIGP,SIGA,SIGB,D,PHI,QMX,QMN, THETA, PI,WKAREA(100),
     * HOLANG, SANG
      INTEGER I,NUM,K ,HOLD,IER,EIR,IWK(2),RUN,IDGT,M
C
C
C READ IN NUMBER OF STATIONS TO BE USED IN COMPUTATION.
      NUM=4
C
```

```
C     INPUT THE X AND Y COORDINATES OF THE CONTROL STATIONS (MTM)
C
      XSTA(1)=7974.86D0
      XSTA(2)=6978.19D0
      XSTA(3)=5598.98D0
      XSTA(4)=4371.50D0
      YSTA(1)=3909.43D0
      YSTA(2)=2828.77D0
      YSTA(3)=1982.76D0
      YSTA(4)=2840.28D0
C
C    INPUT THE STANDARD ERROR OF EACH RANGE
C
      SIGMA(1) = 3.0D0
      SIGMA(2) = 3.0D0
      SIGMA(3) = 3.0D0
      SIGMA(4) = 3.0D0
C
C     INPUT THE BASE-LINE CORRECTOR FOR EACH RANGE
C
      COR(1) = +0.27D0
      COR(2) = -1.50D0
      COR(3) = -0.22D0
      COR(4) = -4.37D0
C
C READ IN RANGE DATA FROM DATA FILE
C
  21  CONTINUE
      READ(4,18) IFIX,RANGE(4),RANGE(3),RANGE(2),RANGE(1)
C
C     SET UP SENTINNEL VALUES TO STOP READING FILE
C
      IF(IFIX.EQ.9999) GO TO 900
      IF(RANGE(1).GT.8999.9) GO TO 21
      IF(RANGE(2).GT.8999.9) GO TO 21
      IF(RANGE(3).GT.8999.9) GO TO 21
      IF(RANGE(4).GT.8999.9) GO TO 21
```

```
C
C   CORRECT OBSERVED RANGES FOR BASE-LINE CORRECTORS
C
      RANGE(1) = RANGE(1) + COR(1)
      RANGE(2) = RANGE(2) + COR(2)
      RANGE(3) = RANGE(3) + COR(3)
      RANGE(4) = RANGE(4) + COR(4)
C
 18   FORMAT(I4,6X,F10.2,F10.2,F10.2,F10.2)
      DO 35 I=1,NUM
         XOSTA(I)=XSTA(I)
         YOSTA(I)=YSTA(I)
         RANGEO(I)=RANGE(I)
 35   CONTINUE
C
C
C
C BEGIN ROUTINE TO DETERMINE BEST STATION PAIR FOR ESTIMATE
C
C
C
C     GO TO 800
          ADJ=YSTA(1) - YSTA(2)
          CALL DIST(XSTA(1),YSTA(1),XSTA(2),YSTA(2), HOLDIS)
          LANG=DARSIN(ADJ/HOLDIS)
          CALL BETA(RANGE(2),RANGE(1),HOLDIS,CCC)
          IF (CCC.GE.LANG) THEN
              FF=CCC-LANG
              DIFX=RANGE(1)*DCOS(FF)
              DIFY=RANGE(1)*DSIN(FF)
              FINX=XSTA(1)-DIFX
              FINY=YSTA(1)+DIFY
           ELSE
             FF=LANG-CCC
              DIFX=RANGE(1)*DCOS(FF)
              DIFY=RANGE(1)*DSIN(FF)
              FINX=XSTA(1)-DIFX
```

```
              FINY=YSTA(1)-DIFY
            END IF
C
C   COMPUTE THE WEIGHT MATRIX
C
      DO 11 I= 1,NUM
       DO 12 J = 1,NUM
         WT(I,J) = 0.0D0
 12    CONTINUE
 11   CONTINUE
      DO 13 I = 1,NUM
        WT(I,I) = 1.0D0 / (SIGMA(I) * * 2)
 13   CONTINUE
C
C
C
C   BEGIN LEAST SQUARES COMPUTATIONS
C
C   CONSTRUCT B MATRIX(BRAY) BY COMPUTING C1,C2,..CNUM.
C
 800  CONTINUE
      RUN=1
 802    CONTINUE
      DO 100 I=1,NUM
          CRAY(I)=DSQRT((FINX-XSTA(I))**2+(FINY-YSTA(I))**2)
          BRAY(I)=RANGE(I)-CRAY(I)
 100  CONTINUE
C
C
C  CONSTRUCT "A" MATRIX
C
      DO 110 I=1,NUM
         DO 120 J=1,2
            IF(J.EQ.1)ARAY(I,J)=(FINX-XSTA(I))/CRAY(I)
            IF(J.EQ.2)ARAY(I,J)=(FINY-YSTA(I))/CRAY(I)
 120     CONTINUE
 110   CONTINUE
```

```
C
C
C COMPUTE "A TRANSPOSE W," "A TRANSPOSE W A," AND
C "A TRANSPOSE W B"
C
      CALL ATRANB(ARAY,WT,NUM,2,NUM,NUM,NUM,ATW,2,IER)
      CALL VMULFF(ATW,ARAY,2,NUM,2,2,NUM,ATWA,2,IER)
      CALL VMULFF(ATW,BRAY,2,NUM,1,2,NUM,ATWB,2,IER)
C
C  THE LEAST SQUARES POSITION IS DETERMINED VIA THE FOLLOWING
C  SUBROUTINE FROM THE IMSL LIBRARY
C
      CALL AXB (ATWA,2,2,2,ATWB,2,1,0,C,DEL,2,IWK,WK,IER)
      FINX=FINX+DEL(1,1)
      FINY=FINY+DEL(2,1)
      CALL VMULFF(ARAY,DEL,NUM,2,1,NUM,2,NICK,NUM,IER)
C COMPUTE RESIDUALS
      DO 121 I=1,NUM
          VRAY(I)=NICK(I)-BRAY(I)
 121    CONTINUE
C
C  COMPUTE THE INVERSE OF ATWA = VARIANCE-COVARIANCE
C MATRIX = Q
C
C
      IDGT = 0
      CALL LINV2F (ATWA,2,2,Q,IDGT,WKAREA,IER)
C
C  COMPUTE STANDARD DEVIATION OF UNIT WEIGHT = SIGMA NAUGHT
C = SIGNO = SQRT(VTWV / N - M)
C
      M = 2
      CALL ATRANB (VRAY,WT,NUM,1,NUM,NUM,NUM,VTW,1,IER)
      CALL VMULFF (VTW,VRAY,1,NUM,1,1,NUM,VTWV,1,IER)
      SIGNO = DSQRT( VTWV(1,1) / (NUM - M))
C   END STD DEVIATION OF UNIT WEIGHT
C
```

```
C   COMPUTE STANDARD DEVIATION OF X,Y, AND COVARIANCE = SIGX,
C SIGY, AND SIGXY
      SIGX = SIGNO * DSQRT(Q(1,1))
      SIGY = SIGNO * DSQRT(Q(2,2))
      SIGXY = ((SIGNO * SIGNO) * Q(1,2))
C
C  END SIGMAX, SIGMAY, SIGMAXY
C
C   COMPUTE SIGMA P = SQRT( (SIGX * * 2) + (SIGY * * 2) )
C
      SIGP = DSQRT( (SIGX * *2) + (SIGY * * 2) )
C
C   END SIGMA P
C
C  COMPUTE CORRELATION COEFFICIENT = RHO
C
      RHO = SIGXY / (SIGX * SIGY)
C
C END CORRELATION COEFF.
C
C
C  COMPUTE ERROR ELIPSE
C
C    COMPUTE SIGMA A = SEMI-MAJOR AXES, SIGMA B = SEMI-MINOR
C AXES AND THETA = ORIENTATION ANGLE
C
      D = DSQRT((Q(1,1) -Q(2,2)) * * 2 + 4.0D0*(Q(1,2)* * 2))
      QMX = ((Q(1,1) + Q(2,2)) / 2.0D0) + (D / 2.0D0)
      QMN = ((Q(1,1) + Q(2,2)) / 2.0D0) - (D / 2.0D0)
C
C     SIGA = SIGNO * DSQRT(QMX)
      SIGA = SIGNO * DSQRT((2.0D0 * Q(1,1) * Q(2,2)) /
C    # (Q(1,1) + Q(2,2)  - D))
C
C     SIGB = SIGNO * DSQRT(QMN)
      SIGB = SIGNO * DSQRT((2.0D0 * Q(1,1) * Q(2,2)) /
C    #(Q(1,1) + Q(2,2) + D))
```

```
C
         PHI = (2.0D0 * Q(1,2)) / (Q(1,1) - Q(2,2))
         THETA = 0.5D0 * DATAN (PHI)
         PI = DARCOS (-1.0D0)
         THETA = THETA * ( 180.0D0 / PI)
C
C
      RUN=RUN+1
      IF(RUN.EQ.15) GO TO 900
      IF((DABS(DEL(1,1))).LT.(0.1D0))THEN
         IF((DABS(DEL(2,1))).LT.(0.1D0))THEN
            GO TO 900
         END IF
      END IF
C     WRITE(6,901)
      GO TO 802
  900 CONTINUE
      go to 21
      STOP
      END
      SUBROUTINE DIST(X1,Y1,X2,Y2,ANS)
         REAL*8 X1,Y1,X2,Y2,ANS
         ANS=DSQRT((X1-X2)**2+(Y1-Y2)**2)
      RETURN
      END
      SUBROUTINE BETA (A,B,C,BET)
         REAL*8 A,B,C,BET
         BET=DARCOS ((B**2+C**2-A**2)/(2.D0*B*C))
      RETURN
      END
C   IMSL ROUTINE NAME:   LLBQF
C
C-----------------------------------------------------------------------
C
C   COMPUTER:                 IBM/DOUBLE
C
C   LATEST REVISION:          JUNE 1, 1980
```

```
C
C   PURPOSE:            SOLUTION OF LINEAR LEAST SQUARES PROBLEM -
C                           HIGH ACCURACY SOLUTION
C
C   USAGE:            CALL LLBQF (A,IA,M,N,B,IB,NB,IND,C,X,IX,IWK,
C                           WK,IER)
C-------------------------------------------------------------------
C
      SUBROUTINE AXB   (A,IA,M,N,B,IB,NB,IND,C,X,IX,IWK,WK,IER)
C
C                                 SPECIFICATIONS FOR ARGUMENTS
      INTEGER            IA,M,N,IB,NB,IND,IX,IER,IWK(1)
      DOUBLE PRECISION   A(IA,N),B(IB,NB),C(4),X(IX,NB),WK(1)
C                              SPECIFICATIONS FOR LOCAL VARIABLES
      INTEGER          D,IFAIL,IFF,IQRIJ,IQR,IRES,IV,IY,I,J,K,MPN,
     1                  NP1,M1,N1
      LOGICAL            BASIC
      DOUBLE PRECISION   TOL
C                                   FIRST EXECUTABLE STATEMENT
      IER = 129
      IF ((N.LE.0).OR.(M.LE.0)) GO TO 9000
C                                   SETUP
      IF (IND.EQ.1) GO TO 10
      DO 5 I=1,4
    5 C(I) = 0.0D0
   10 CONTINUE
      M1 = C(1)
      IF (M1.LT.0.OR.M1.GT.N) M1 = 0
      TOL = DABS(C(2))
      BASIC = C(3).NE.1.0D0
      MPN = M+N
      NP1 = N+1
      IRES = 1
      ID = IRES+M
      IQR = ID+N
      IFF = IQR+MPN*NP1
      IY = IFF+MPN
```

```
      IQRIJ = IQR
      DO 25 J=1,N
         DO 15 K=1,NB
15       X(J,K) = 0.0D0
         DO 20 I=1,M
            WK(IQRIJ) = A(I,J)
            IQRIJ = IQRIJ+1
20       CONTINUE
         IQRIJ = IQRIJ+N
25    CONTINUE
      DO 30 I=1,M
         WK(IQRIJ) = B(I,1)
         IQRIJ = IQRIJ+1
30    CONTINUE
C                                               QR DECOMPOSITION OF A
      CALL LLBQG (WK(IQR),MPN,NP1,M,N,M1,N1,BASIC,TOL,
     # IWK,WK(ID))
      C(4) = N1
      IF (N1.EQ.0) GO TO 9005
      IER = 130
      IF (N1.LT.M1) GO TO 9000
      IER = 0
      IFAIL = 0
C                                           SOLVE NB RIGHT-HAND-SIDES
      DO 35 IV=1,NB
35    CALL LLBQH (A,IA,M,N,M1.N1,B(1,IV),WK(IQR),MPN,BASIC,
     1X(1,IV),IWK,WIK(IRES),WK(ID),WK(IFF),WK(IY),IFAIL)
      IF (IFAIL.EQ.0) GO TO 9005
      IER = 131
 9000 CONTINUE
      CALL UERTST (IER,6HLLBQF )
 9005 RETURN
      END
```

APPENDIX C

FOUR-RANGE MLOP FIX ACCURACIES

EXAMPLE DATA OUTPUT FROM PROGRAM SLEAST

POSITION FROM FOUR RANGES

CONTROL STATIONS WERE SQUARE, CONK, USE MON, GEOCEIVER

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
1	0.90	1.13	0.48	1.43	0.81	-31.86	1.45
2	1.34	1.62	0.89	1.96	1.22	-32.67	2.10
3	1.39	1.62	0.76	1.87	1.28	-32.87	2.14
4	1.56	1.79	0.74	1.98	1.46	-31.53	2.37
9	0.31	0.39	0.00	0.39	0.31	-2.70	0.50
10	0.76	0.97	-0.00	0.97	0.76	0.30	1.23
11	0.59	0.76	-0.01	0.77	0.59	2.36	0.96
13	0.71	0.95	-0.04	0.96	0.71	5.69	1.19
14	0.59	0.80	-0.04	0.80	0.59	7.42	0.99
15	0.59	0.80	-0.05	0.81	0.59	9.04	1.00
17	0.32	0.45	-0.02	0.46	0.32	12.10	0.56
18	0.47	0.67	-0.06	0.69	0.46	13.39	0.82
20	0.94	1.39	-0.31	1.46	0.92	15.31	1.68
21	1.09	1.25	-0.69	1.67	0.94	37.36	1.66
22	1.15	1.28	-0.64	1.63	1.01	38.02	1.72
23	1.70	1.84	-1.14	2.23	1.51	38.71	2.51
24	1.69	1.79	-0.92	2.09	1.52	39.40	2.46

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
25	0.98	1.02	-0.25	1.16	0.89	40.16	1.41
27	1.12	1.14	-0.20	1.23	1.04	41.68	1.59
30	0.89	0.89	-0.05	0.92	0.86	43.70	1.26
31	0.93	0.92	-0.03	0.94	0.91	-41.30	1.31
32	0.19	0.19	-0.00	0.19	0.19	-20.30	0.27
33	0.86	0.83	0.01	0.86	0.83	14.68	1.20
34	0.79	0.76	0.02	0.79	0.75	25.32	1.09
35	0.96	0.93	0.05	0.97	0.92	33.29	1.34
36	0.88	0.87	0.05	0.91	0.85	38.79	1.24
37	0.87	0.86	0.06	0.90	0.83	42.80	1.22
38	0.57	0.56	0.03	0.60	0.54	43.67	0.80
39	0.87	0.86	0.10	0.93	0.82	42.84	1.23
40	0.50	0.50	0.04	0.54	0.46	43.18	0.70
41	0.58	0.58	0.06	0.64	0.54	44.14	0.82
42	0.72	0.72	0.10	0.80	0.65	44.88	1.01
43	0.67	0.67	0.10	0.76	0.60	44.92	0.94
51	0.32	0.35	0.02	0.37	0.30	-32.03	0.47
53	0.84	0.93	0.09	0.96	0.82	-22.93	1.25
54	0.64	0.72	0.04	0.73	0.63	-17.40	0.96
56	0.32	0.37	0.00	0.37	0.32	-5.92	0.49
57	0.36	0.42	0.00	0.42	0.36	-2.39	0.55

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
58	0.53	0.63	-0.00	0.63	0.53	1.44	0.82
61	0.51	0.62	-0.02	0.62	0.51	9.89	0.80
62	0.45	0.55	-0.02	0.56	0.45	11.76	0.71
63	0.51	0.62	-0.03	0.63	0.51	13.62	0.81
67	1.80	2.20	-0.71	2.30	1.75	20.89	2.85
69	1.06	1.30	-0.31	1.39	1.02	23.69	1.67
70	0.29	0.35	-0.03	0.38	0.28	26.28	0.46
71	0.36	0.43	-0.05	0.48	0.34	30.20	0.56
72	0.81	0.94	-0.27	1.10	0.74	33.95	1.24
73	1.14	1.29	-0.63	1.62	1.01	36.58	1.72
74	0.83	0.94	-0.39	1.26	0.71	38.07	1.25
75	0.75	0.86	-0.39	1.28	0.63	38.51	1.14

APPENDIX D

FOUR-AZIMUTH MLOP FIX ACCURACIES

DATA PROCESSED USING A MODIFICATION OF PROGRAM SILVA1 FROM SILVA (1982) TO HANDLE LARGE DATA FILES.

EXAMPLE DATA OUTPUT FROM PROGRAM LSQAZ4.

POSITION FROM FOUR AZIMUTHS.

CONTROL STATIONS WERE SQUARE, CONK, USE MON, GEOCEIVER.

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
76	0.08	0.15	-0.00	0.16	0.08	97.59	0.17
77	0.04	0.07	0.00	0.07	0.04	90.12	0.08
78	0.61	0.92	0.05	0.92	0.61	83.75	1.10
79	0.31	0.41	0.02	0.41	0.30	78.10	0.51
80	0.23	0.28	0.01	0.29	0.23	74.24	0.37
81	0.80	0.88	0.05	0.89	0.79	71.48	1.19
82	0.43	0.45	0.00	0.45	0.43	76.41	0.62
83	0.42	0.43	-0.01	0.43	0.42	129.29	0.60
84	0.51	0.51	-0.02	0.53	0.49	137.63	0.72
85	0.25	0.25	-0.01	0.27	0.24	135.85	0.36
86	0.33	0.33	-0.02	0.36	0.31	132.84	0.47
87	0.39	0.40	-0.02	0.42	0.37	131.27	0.56
88	0.56	0.55	-0.03	0.59	0.53	135.74	0.78
89	0.43	0.40	-0.01	0.44	0.39	158.24	0.59
90	1.22	0.99	-0.09	1.23	0.99	170.41	1.57

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
91	0.58	0.55	-0.10	0.68	0.50	140.02	0.80
92	0.20	0.19	-0.01	0.23	0.18	135.75	0.28
93	0.73	0.75	-0.12	0.83	0.67	131.11	1.04
94	0.58	0.61	-0.07	0.66	0.54	127.81	0.84
95	0.61	0.65	-0.07	0.70	0.58	125.75	0.89
96	0.44	0.47	-0.03	0.50	0.42	122.77	0.65
98	0.33	0.36	-0.01	0.37	0.32	114.39	0.49
99	0.51	0.57	-0.02	0.58	0.50	104.35	0.76
100	0.39	0.45	-0.01	0.46	0.39	95.63	0.60
101	0.39	0.49	-0.00	0.49	0.39	91.62	0.63
103	0.76	1.11	-0.04	1.11	0.76	93.19	1.34
104	0.20	0.32	-0.01	0.32	0.20	96.65	0.38
106	0.32	0.52	-0.06	0.57	0.31	106.55	0.61
107	0.28	0.44	-0.03	0.46	0.27	103.64	0.52
108	0.70	1.06	-0.13	1.09	0.69	101.25	1.27
111	0.61	0.79	-0.[illegible]	0.80	0.60	102.03	0.99
112	0.54	0.67	-0.05	0.68	0.53	105.32	0.86
113	1.47	1.77	-0.39	1.83	1.44	109.54	2.30
114	0.53	0.62	-0.06	0.65	0.51	113.55	0.81
115	1.00	1.16	-0.24	1.24	0.96	116.71	1.54
116	0.60	0.69	-0.10	0.75	0.57	119.17	0.91
117	0.74	0.85	-0.17	0.94	0.69	121.17	1.13
118	0.71	0.81	-0.17	0.92	0.66	122.83	1.08
119	0.56	0.65	-0.12	0.75	0.51	123.84	0.86
120	0.45	0.68	-0.12	0.77	0.42	111.38	0.81

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
122	1.44	1.95	-0.97	2.20	1.37	114.13	2.43
124	0.41	0.53	-0.06	0.57	0.39	113.84	0.66
125	0.53	0.68	-0.10	0.74	0.51	113.00	0.86
126	0.67	0.87	-0.14	0.92	0.65	111.57	1.10
127	0.50	0.65	-0.07	0.69	0.49	109.83	0.82
128	0.51	0.67	-0.07	0.70	0.50	107.96	0.85
129	0.45	0.61	-0.05	0.63	0.44	106.26	0.75
131	0.36	0.52	-0.04	0.54	0.35	104.58	0.63
133	1.03	1.60	-0.47	1.72	1.00	105.07	1.90
134	0.31	0.50	-0.05	0.55	0.30	107.91	0.58
136	0.44	0.72	-0.14	0.84	0.42	110.70	0.85
137	0.61	0.98	-0.23	1.10	0.59	108.89	1.15
138	0.76	1.19	-0.30	1.31	0.74	107.63	1.42
139	0.79	1.21	-0.28	1.30	0.77	106.81	1.44
141	0.27	0.40	-0.03	0.42	0.26	106.77	0.48
142	0.34	0.49	-0.04	0.52	0.33	107.28	0.60
143	0.51	0.73	-0.10	0.77	0.49	107.89	0.88
145	1.12	1.63	-0.53	1.76	1.09	108.65	1.98
147	1.08	1.67	-0.57	1.82	1.04	107.68	1.98
148	0.88	1.45	-0.42	1.58	0.86	106.30	1.70
150	0.89	1.30	-0.44	1.48	0.85	112.26	1.58
151	0.72	0.92	-0.26	1.09	0.67	119.01	1.17
152	0.56	0.62	-0.13	0.75	0.50	127.65	0.84
154	0.69	0.60	-0.07	0.73	0.58	155.12	0.92
155	0.72	0.62	0.01	0.72	0.62	3.15	0.95

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
156	0.37	0.26	-0.05	0.47	0.24	151.08	0.45
157	0.40	0.38	-0.06	0.51	0.33	138.38	0.55
158	0.87	0.92	-0.23	1.06	0.78	129.72	1.26
159	0.23	0.25	-0.01	0.26	0.22	124.69	0.33
160	0.39	0.40	-0.02	0.43	0.37	127.75	0.56
161	0.40	0.41	-0.02	0.44	0.38	131.07	0.57
163	0.40	0.42	-0.04	0.46	0.37	127.67	0.58
164	0.59	0.65	-0.10	0.72	0.55	123.37	0.88
165	0.29	0.35	-0.03	0.38	0.27	118.55	0.45
166	0.67	0.86	-0.17	0.95	0.63	114.46	1.09
167	0.96	1.70	-0.51	1.82	0.94	103.91	1.95
168	0.94	1.55	-0.44	1.67	0.92	105.10	1.81
170	1.64	2.36	-1.05	2.53	1.60	108.11	2.88
171	0.78	1.05	-0.20	1.11	0.76	109.91	1.30
172	1.01	1.26	-0.28	1.34	0.97	111.85	1.62
174	1.81	2.01	-0.57	2.11	1.75	118.04	2.71
175	0.23	0.24	-0.01	0.25	0.22	124.17	0.33
175	0.23	0.24	-0.01	0.25	0.22	124.17	0.33
177	1.07	1.01	-0.11	1.10	0.98	150.74	1.47
178	1.41	1.24	-0.17	1.44	1.23	161.06	1.88
179	0.22	0.18	-0.00	0.22	0.18	168.43	0.28

APPENDIX E

FORTRAN PROGRAM FOR LEAST SQUARES POSITION FROM HYBRID METHODS

```
C PROGRAM R2+AZ2:  RUN IN VS FORTRAN OR WATFIV
C
C
C THE PROGRAM IS A MODIFICATION OF PROGRAM SILVA3 (SILVA, 1982) IN
C COMBINATION WITH PROGRAM RR+AZ (DE BOW,1985). PROGRAM WILL TAKE MLOP'S
C IN THE FORM OF TWO RANGES AND TWO AZIMUTHS AND COMPUTE THE
C LEAST SQUARES POSITION.
C  INPUT THE STATIONS COORDINATES AND STD. ERROR VIA:
C   FILEDEF 01: STATION X, Y AND SIGMA.
C
C FOR CONVENTION, LET:
C   S1 COINCIDE WITH S3(A RANGE DISTANCE IS OBSERVED FROM S1)
C   S2 COINCIDE WITH S4(A RANGE DISTANCE IS OBSERVED FROM S2)
C
C INPUT THE RANGE DISTANCES AND HORIZONTAL ANGLES OBSERVED
C   USING FORMAT 260 AND:
C   FILEDEF 02:  RANGE 1, RANGE 2, AZ1, AZ2, POS #
C
C WHEN NO MORE DATA ARE AVAILABLE, INPUT A 'DUMMY' DATA SET WITH
C  ALL VALUES EQUAL TO 0.0 USING FORMAT 260
C
C SPECIFIED OUTPUT WILL APPEAR ON:
C FILEDEF 03 : DATA OUTPUT FILE
C
C  #### THE BASE-LINE CORRECTORS MUST BE CHANGED FOR DIFFERENT
C       STATION COMBINATIONS BY CHANGING THE ORDER OF COR(1),
C       COR(2),ETC.
C
C  ****** NOTE******
C  PROGRAM " CHANGE3 FORTRAN B1" WILL OUTPUT THE RANGES AND
C  AZIMUTHS IN THE DESIRED ORDER AND ALSO OMIT BAD RANCES
C  AND AZIMUTHS!!!!
```

```
C  ***********************
C
C POSITION FROM TWO RANGES AND TWO AZIMUTHs
      INTEGER N,I,J,K,IFIX,POS
      REAL*8 PI,TB(10,4),E,E1,E2,E3,E4,XO,YO,X01,Y01,X02,Y02,
     1   A301,A302,F,DSQRT,A30,A40,S10,S20,S30,S40,A(10,2),DSIN,
     2   DCOS, L(10),A303,X(2),AX(10),V(10),VTW(10),VTWV,TRACE,
     3   CHARLE,SU,SX,SY,SXY,A304,RO,D,SA,SB,GAMA,OMEGA,X10,X11,
     4   X1,Y1,AVER,D1,GAMAO,DARSIN,S,TBW(10,10),GREAT,ATW(2,10),
     5   ATWA(2,2),Q(2,2),BETA,ATWL(2),DELTAX,DELTAY,TOL,DTAN,
      REAL * 8 DATAN,VS(10),DIS(10),COR(10)
      N=4
      PI=DARCOS(-1.0D0)
      WRITE(3,163)
  163 FORMAT('1',1X)
      WRITE(3,124)
  124 FORMAT(/,'POSITION FROM TWO RANGES AND TWO AZIMUTHS',/,1X,
     # 'DATE : ',10X,' DAY :  ',/)
C  READ IN STATION COORDINATES AND SIGMA VALUES
      DO 218 I=1,N
       READ(1,160)TB(I,1),TB(I,2),TB(I,4)
  218 CONTINUE
  160 FORMAT(1X,2F12.2,F9.5)
      DO 219 I=1,N
       WRITE(3,161)I,TB(I,1),TB(I,2),TB(I,4)
  219 CONTINUE
  161 FORMAT('0',1X,'ST#',I2,3X,'X =',F12.2,3X,'Y =',F12.2,3X,
     1       'ST ERROR=',F7.3,2X,'METERS')
C     WRITE(3,128)
C 128 FORMAT(//,' POS #',2X,'X-COORDINATE',2X,'Y-COORDINATE',6X,'V(1)',
C    *10X,'V(2)',10X,'V(3)',10X,'V(4)',/)
      WRITE(3,150)
  150 FORMAT(///,1X,'FIX #',2X,'SIGO',8X,'SIG X',4X,'SIG Y',4X,
     *'SIG P',4X,'SIG XY',4X,'SIG A',3X,'SIG B',4X,'THETA',/)
C IF SIGMA3=TB(3,4) IS IN METERS THE FOLLOWING IS COMMMENTED OUT
C IF SIGMA4=TB(4,4) IS IN METERS THE FOLLOWING IS COMMMENTED OUT
C
```

```
C       CONVERSION DEGREES RADIANS(TB(3,4))
C      TB(3,4)=TB(3,4)*(PI/180.0)
C      TB(4,4)=TB(3,4)*(PI/180.0)
C
C  INPUT OBSERVED RANGES R1 AND R2 IN METERS , OBSERVED AZIMUTH
C        AZ1 AND AZ2 IN DEGREES AND POSITION NUMBER
C
 500  CONTINUE
      READ(2,260)TB(1,3),TB(2,3),TB(3,3),Tb(4,3), POS
C
  260 FORMAT(1X,2(F12.2,1X),2(F9.5,1X),I4)
C
C     APPLY CALIBRATION CORRECTORS TO OBSERVED RANGES
C
      COR(1) = +0.27D0
      COR(2) = -1.50D0
C     COR(2) = -0.22D0
C     COR(2) = -4.37D0
      TB(1,3) = TB(1,3)  + COR(1)
      TB(2,3) = TB(2,3) + COR(2)
C    SET UP SENTINEL VALUES TO STOP READING FILE
  220 IF(TB(1,3).EQ.0.0)GO TO 999
  220 IF(POS.EQ.9999) GO TO 999
C   CONVERSION DEGREES RADIANS(TB(3,3))
        TB(3,3)=TB(3,3)*(PI/180.0)
        TB(4,3)=TB(4,3)*(PI/180.0)
C START ROUTINE TO DETERMINE FIRST INITIAL POINT
      E=TB(1,3)**2-TB(2,3)**2+TB(2,2)**2-TB(1,2)**2+
     1  TB(2,1)**2-TB(1,1)**2
      IF(TB(2,1).EQ.TB(1,1))GO TO 227
       E1=((TB(1,2)-TB(2,2))/(TB(2,1)-TB(1,1)))**2+1.0
       E2=(E*(TB(1,2)-TB(2,2)))/((TB(2,1)-TB(1,1))**2)-
     2    2.0*TB(1,1)*((TB(1,2)-TB(2,2))/
     3    (TB(2,1)-TB(1,1)))-2.0*TB(1,2)
       E3=(E/(2.0*(TB(2,1)-TB(1,1))))**2-(E*TB(1,2))/
     4    (TB(2,1)-TB(1,1))-TB(1,3)**2+TB(1,1)**2+
     5    TB(1,2)**2
```

```
      E4=E2**2-4.0*E1*E3
      IF(E4.GE.0.0)GO TO 228
       XO=TB(1,1)+TB(1,3)*(TB(2,1)-TB(1,1))/DSQRT
     6    ((TB(2,1)-TB(1,1))**2+(TB(2,2)-TB(1,2))**2)
       YO=TB(1,2)+TB(1,3)*(TB(2,2)-TB(1,2))/DSQRT
     7    ((TB(2,1)-TB(1,1))**2+(TB(2,2)-TB(1,2))**2)
      GO TO 229
228   IF(E4.NE.0.0)GO TO 230
       YO=E2/(-2.0*E1)
       XO=E/(2.0*(TB(2,1)-TB(1,1)))+YO*(TB(1,2)-TB(2,2))/
     8    (TB(2,1)-TB(1,1))
      GO TO 229
230   CONTINUE
       Y01=(-E2+DSQRT(E4))/(2.0*E1)
       X01=E/(2.0*(TB(2,1)-TB(1,1)))+Y01*(TB(1,2)-TB(2,2))/
     1     (TB(2,1)-TB(1,1))
       Y02=(-E2-DSQRT(E4))/(2.0*E1)
       X02=E/(2.0*(TB(2,1)-TB(1,1)))+Y02*(TB(1,2)-TB(2,2))/
     2     (TB(2,1)-TB(1,1))
       IF(TB(3,1).NE.X01.OR.TB(3,2).NE.Y01)GO TO 231
       IF(TB(4,1).NE.X01.OR.TB(4,2).NE.Y01)GO TO 231
        XO=X02
        YO=Y02
       GO TO 232
231    IF(TB(3,1).NE.X02.OR.TB(3,2).NE.Y02)GO TO 233
       IF(TB(4,1).NE.X02.OR.TB(4,2).NE.Y02)GO TO 233
        XO=X01
        YO=Y01
       GO TO 232
233    CONTINUE
        CALL CHANGE(TB(3,1),TB(3,2),X01,Y01,A301)
        CALL CHANGE(TB(3,1),TB(3,2),X02,Y02,A302)
        CALL CHANGE(TB(4,1),TB(4,2),X01,Y01,A303)
        CALL CHANGE(TB(4,1),TB(4,2),X02,Y02,A304)
        IF(A301.NE.TB(3,3).OR.A301.NE.A302)GO TO 234
        IF(A303.NE.TB(4,3).OR.A303.NE.A304)GO TO 234
         WRITE(3,167)
```

```
167 FORMAT(/////,1X,'SOLUTION UNDETERMINED FOR THAT DATA SET')
       GO TO 998
234    IF(((A301-TB(3,3))**2).NE.((A302-TB(3,3))**2).OR.((A303
   #   -TB(4,3))**2).NE.((A304-TB(4,3))**2))GO TO 235
        XO=(X01+X02)/2.0d0
        YO=(Y01+Y02)/2.0d0
       GO TO 236
235    IF(((A301-TB(3,3))**2).LE.((A302-TB(3,3))**2).OR.((A303
   #   -TB(4,3))**2).LE.((A304-TB(4,3))**2))GO TO 237
        XO=X02
        YO=Y02
       GO TO 236
237    IF(((A301-TB(3,3))**2).GE.((A302-TB(3,3))**2).OR.((A303
   #   -TB(4,3))**2).GE.((A304-TB(4,3))**2))GO TO 236
        XO=X01
        YO=Y01
236    CONTINUE
232   CONTINUE
229  CONTINUE
    GO TO 238
227 CONTINUE
     YO=E/(2.0*(TB(2,2)-TB(1,2)))
     F=TB(1,3)**2-(TB(1,2)-YO)**2
     IF(F.GT.0.0D0)GO TO 239
      XO=TB(1,1)
     GO TO 240
239  CONTINUE
      X01=TB(1,1)+DSQRT(F)
      Y01=YO
      X02=TB(1,1)-DSQRT(F)
      Y02=YO
      IF(TB(3,1).NE.X01.OR.TB(3,2).NE.Y01)GO TO 241
       XO=X02
      GO TO 242
241   IF(TB(3,1).NE.X02.OR.TB(3,2).NE.Y02)GO TO 243
       XO=X01
      GO TO 242
```

```
 243    CONTINUE
         CALL CHANGE(TB(3,1),TB(3,2),X01,Y01,A301)
         CALL CHANGE(TB(3,1),TB(3,2),X02,Y02,A302)
         CALL CHANGE(TB(4,1),TB(4,2),X01,Y01,A303)
         CALL CHANGE(TB(3,1),TB(3,2),X02,Y02,A304)
         IF(A301.NE.TB(3,3).OR.A301.NE.A302)GO TO 244
         IF(A303.NE.TB(4,3).OR.A303.NE.A304)GO TO 244
          WRITE(3,167)
         GO TO 998
 244     IF(((A301-TB(3,3))**2).NE.((A302-TB(3,3))**2).OR.((A303
    #    -TB(4,3))**2).NE.((A304-TB(4,3))**2))GO TO 245
          XO=X01
         GO TO 246
 245     IF(((A301-TB(3,3))**2).LE.((A302-TB(3,3))**2).OR.((A303
    #    -TB(4,3))**2).LE.((A304-TB(4,3))**2))GO TO 247
          XO=X02
         GO TO 246
 247     IF(((A301-TB(3,3))**2).GE.((A302-TB(3,3))**2).OR.((A301
    #    -TB(4,3))**2).GE.((A302-TB(4,3))**2))GO TO 246
          XO=X01
 246     CONTINUE
 242    CONTINUE
 240   CONTINUE
 238  CONTINUE
C   ITERATIONS(TOL)
  51  CONTINUE
C   A30(TB,XO,YO)
      CALL CHANGE(TB(3,1),TB(3,2),XO,YO,A30)
      CALL CHANGE(TB(4,1),TB(4,2),XO,YO,A40)
C   DISTANCES(TB,XO,YO)
      S10=DSQRT((TB(1,1)-XO)**2+(TB(1,2)-YO)**2)
      S20=DSQRT((TB(2,1)-XO)**2+(TB(2,2)-YO)**2)
      S30=DSQRT((TB(3,1)-XO)**2+(TB(3,2)-YO)**2)
      S40=DSQRT((TB(4,1)-XO)**2+(TB(4,2)-YO)**2)
C   MATRIX A(TB,XO,YO,S10,S20,S30,A30,A40)
       DO 38 I=1,10
        DO 39 J=1,2
```

```
          A(I,J)=0.0D0
   39    CONTINUE
   38   CONTINUE
        A(1,1)=(XO-TB(1,1))/S10
        A(1,2)=(YO-TB(1,2))/S10
        A(2,1)=(XO-TB(2,1))/S20
        A(2,2)=(YO-TB(2,2))/S20
        A(3,1)=DCOS(A30-TB(3,3))*(YO-TB(3,2))/S30+
       1       DSIN(A30-TB(3,3))*(XO-TB(3,1))/S30
        A(3,2)=DCOS(A30-TB(3,3))*(TB(3,1)-XO)/S30+
       2       DSIN(A30-TB(3,3))*(YO-TB(4,2))/S30
        A(4,1)=DCOS(A40-TB(4,3))*(YO-TB(4,2))/S40+
       1       DSIN(A40-TB(4,3))*(XO-TB(4,1))/S40
        A(4,2)=DCOS(A40-TB(4,3))*(TB(4,1)-XO)/S40+
       2       DSIN(A40-TB(4,3))*(YO-TB(4,2))/S40
C
C    LIST L(TB,S10,S20,S30,A30)
         DO 34 I=1,10
          L(I)=0.0D0
   34   CONTINUE
        L(1)=TB(1,3)-S10
        L(2)=TB(2,3)-S20
        L(3)=DSIN(TB(3,3)-A30)*S30
        L(4)=DSIN(TB(4,3)-A40)*S40
C
C   THE FOLLOWING IS COMMENTED OUT IF TB(3,4) IS IN METERS!!!
C       TB(3,4)=S30*DSIN(TB(3,4))
C       TB(4,4)=S40*DSIN(TB(4,4))
C
C    WEIGHT MATRIX(N,TB(I,4))
         DO 11 I=1,10
          DO 12 J=1,10
           TBW(I,J)=0.0D0
   12    CONTINUE
   11   CONTINUE
C
C         SQUARE(N,TB(I,4),TBW)
```

```
      DO 13 I=1,N
       TBW(I,I)=TB(I,4)**2
   13 CONTINUE
C
C      NORMALIZE(TBW)
      GREAT=TBW(1,1)
      DO 14 I=2,N
       IF(TBW(I,I).GT.GREAT)GREAT=TBW(I,I)
   14 CONTINUE
      DO 15 I=1,N
       TBW(I,I)=GREAT/TBW(I,I)
   15 CONTINUE
C
C   AFTER WEIGHT MATRIX(TB(3,4),S30)
C     TB(3,4)=DARSIN(TB(3,4)/S30)
C
C    NORMAL EQUATIONS(A,TBW,L)
C   TRANSPOSE(A)*TBW(A,TBW)
     DO 43 I=1,2
      DO 44 J=1,10
       ATW(I,J)=0.0D0
       DO 45 K=1,10
        ATW(I,J)=ATW(I,J)+A(K,I)*TBW(K,J)
   45  CONTINUE
   44 CONTINUE
   43 CONTINUE
C
C   MATRIX ATWA(ATW,A)
     DO 46 I=1,2
      DO 47 J=1,2
       ATWA(I,J)=0.0D0
       DO 48 K=1,10
        ATWA(I,J)=ATWA(I,J)+ATW(I,K)*A(K,J)
   48  CONTINUE
   47 CONTINUE
   46 CONTINUE
C
```

```
C   INVERT ATWA(ATWA)
      BETA=ATWA(1,2)**2-ATWA(1,1)*ATWA(2,2)
      Q(1,1)=-ATWA(2,2)/BETA
      Q(1,2)=ATWA(1,2)/BETA
      Q(2,1)=Q(1,2)
      Q(2,2)=-ATWA(1,1)/BETA
C
C   MATRIX ATWL(ATWL)
      DO 49 I=1,2
       ATWL(I)=0.0D0
       DO 50 K=1,10
        ATWL(I)=ATWL(I)+ATW(I,K)*L(K)
   50  CONTINUE
   49 CONTINUE
C
C   ADJUSTED INCREMENTS(Q,ATWL)
      DELTAX=Q(1,1)*ATWL(1)+Q(1,2)*ATWL(2)
      DELTAY=Q(2,1)*ATWL(1)+Q(2,2)*ATWL(2)
C
C   NEW INITIAL POINT(XO,YO,DELTAX,DELTAY)
      XO=XO+DELTAX
      YO=YO+DELTAY
C
C   TOLERANCE(DELTAX,DELTAY)
      TOL=DELTAX**2+DELTAY**2
C
      IF(TOL.GE.1.0d0)GO TO 51
C
C   FINAL ADJUSTED POSITION(XO,YO)
C
C   PRECISION(A,TBW,Q,L,DELTAX,DELTAY,N,S30)
C   RESIDUALS(A,L,DELTAX.DELTAY,N)
      X(1)=DELTAX
      X(2)=DELTAY
C        AX(A,X,N)
      DO 52 I=1,N
       AX(I)=0.0D0
```

```
      DO 53 J=1,2
       AX(I)=AX(I)+A(I,J)*X(J)
 53    CONTINUE
 52   CONTINUE
C     END AX(AX)
C   V(AX,L,N)
      DO 54 I=1,N
       V(I)=AX(I)-L(I)
C
C  THE VALUE OF V(3) IS IN RADIANS, THE FOLLOWING CONVERTS V(3)
c  TO ARC LENGTH IN METERS, VS(I).
C
C     VS(3) = V(3)) * S30
C
C     IF RESIDUAL WANTED IN MINUTES OF ARC TAKE COMMENT OFF THE
C     FOLLOWing CONVERSION OF RESIDUAL FROM RADIANS TO MINUTES OF
C     ARC FOR OUTPUT
C     V (3) = V(3) * 3437.7467707849D0
 54   CONTINUE
C
C
C   ST DEVIATION OF UNIT WEIGHT OBS(V,TBW,N)
C      VTW(V,W,N)
      DO 55 I=1,N
       VTW(I)=V(I)*TBW(I,I)
 55    CONTINUE
C
C      VTWV(VTW,V)
      VTWV=0.0D0
      DO 56 I=1,N
       VTWV=VTWV+VTW(I)*V(I)
 56    CONTINUE
C
C      TRACE(TBW)
      TRACE=0.0D0
      DO 57 I=1,N
       TRACE=TRACE+TBW(I,I)
```

```
   57  CONTINUE
C
C        SO(VTWV,TRACE)
       CHARLE=VTWV/(TRACE-20.0D0)
       SU=DSQRT(CHARLE)
C
C
C    ST DEVIATION OF EACH OBS(SU,TBW,S30)
      DO 248 I=1,N
       S=(SU/DSQRT(TBW(I,I)))
  248 CONTINUE
C     S=DARSIN(SU/(DSQRT(TBW(3,3))*S30))*(180.0/PI)
C
C    ST DEVIATIONS AND COVARIANCE OF X AND Y(SO,Q)
      SX=SU*DSQRT(Q(1,1))
      SY=SU*DSQRT(Q(2,2))
      SXY=(SU**2)*Q(1,2)
C
C    CORRELATION CEFFICIENT(SX,SY,SXY)
      RO=SXY/(SX*SY)
C
C    COMPUTE SIGMA P = SQRT( (SIGX * * 2) + (SIGY * * 2))
C
      SP = DSQRT( (SX * *2) + (SY * * 2) )
C
C
C    ERROR ELIPSE(Q,SU)
C    D(Q)
      D=DSQRT((Q(1,1)-Q(2,2))**2+40.0D0*(Q(1,2)**2))
C
C    SEMI-MAJOR AXIS(SU,Q)
      SA=SU*DSQRT(2.0D0*Q(1,1)*Q(2,2)/(Q(1,1)+Q(2,2)-D))
C    SEMI-MINOR AXIS(SU,Q)
      SB=SU*DSQRT(2.0D0*Q(1,1)*Q(2,2)/(Q(1,1)+Q(2,2)+D))
C    GAMA(Q)
      IF(Q(1,1).NE.Q(2,2))GO TO 59
        GAMA=PI/40.0D0
```

```
      GO TO 60
   59  CONTINUE
        OMEGA=DATAN(2.0D0*Q(1,2)/(Q(1,1)-Q(2,2)))
        IF(OMEGA.LT.0.0D0)GO TO 61
         GAMA=OMEGA/20.0D0
        GO TO 62
   61   CONTINUE
          GAMA=  (OMEGA+PI)/2.0D0
   62   CONTINUE
   60 CONTINUE
C
C    INTERSECTION(SU,Q,GAMA)
      X10=(SU**2)*Q(1,1)*Q(2,2)
      X11=Q(2,2)-2.0d0*Q(1,2)*DTAN(GAMA)+(DTAN(GAMA)**2)*Q(1,1)
      X1=X10/X11
      Y1=(DTAN(GAMA)**2)*X1
C
C    AVERAGE(SA,SB)
      AVER=((SA+SB)/2.0d0)**2
C
C    SELECTION(AVER,X1,Y1)
      D1=X1+Y1
      IF(D1.LT.AVER)GO TO 63
        GAMAO=GAMA
      GO TO 64
   63 CONTINUE
        GAMAO=GAMA+PI/20.0D0
   64 CONTINUE
      GAMAO=GAMAO*(180.0D0/PI)
C
C     WRITE(3,129)POSNO,XO,YO,V (1),V (2),V (3),V(4)
C 129 FORMAT(/,1X,I4,2(6X,F8.2),4(3X,F9.4))
      WRITE(3,151)POSNO,SU,SX,SY,SP,SXY,SA,SB,GAMAO
  151 FORMAT(1X,I4,1X,F10.5,6(1X,F8.3),1X,F9.5)
  998  CONTINUE
       READ(2,260)TB(1,3),TB(2,3),TB(3,3),TB(4,3),POSNO
C        (INTRODUCE NEW DATA SET)
```

```
      GO TO 220
  999 CONTINUE
C
      STOP
      END
      SUBROUTINE CHANGE(XS,YS,XP,YP,ASP)
       REAL*8 XS,YS,XP,YP,ASP,PI,ALFA,DATAN
       PI=DARCOS(-1.0D)
       IF(YP.NE.YS.OR.XP.LE.XS)GO TO 221
        ASP=PI/2.0
       GO TO 222
  221  IF(YP.NE.YS.OR.XP.GE.XS)GO TO 223
        ASP=3.0*PI/2.0
       GO TO 222
  223  CONTINUE
        ALFA=DATAN((XP-XS)/(YP-YS))
        IF(ALFA.LT.0.0.OR.XP.LT.XS)GO TO 224
         ASP=ALFA
        GO TO 225
  224   IF(ALFA.GE.0.0.OR.XP.GE.XS)GO TO 226
         ASP=ALFA+2.0*PI
        GO TO 225
  226   CONTINUE
         ASP=ALFA+PI
  225   CONTINUE
  222  CONTINUE
       RETURN
      END
```

APPENDIX F

TWO-RANGE AND TWO-AZIMUTH MLOP FIX ACCURACIES

EXAMPLE DATA OUTPUT FROM PROGRAM R2+AZ2

POSITION FROM 2 RANGES AND 2 AZIMUTHS

CONTROL STATIONS WERE SQUARE, SQUARE ECC., USE MON, USE MON

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
1	1.47	1.66	0.28	1.71	1.44	68.63	2.22
2	1.81	2.04	0.42	2.09	1.78	67.92	2.73
4	1.97	2.16	0.45	2.22	1.93	65.66	2.93
6	0.97	1.04	0.10	1.07	0.95	63.17	1.43
7	0.84	0.89	0.06	0.91	0.82	61.98	1.22
8	0.57	0.60	0.02	0.61	0.56	60.74	0.82
9	0.85	0.88	0.05	0.90	0.84	59.44	1.23
10	0.99	1.01	0.05	1.03	0.97	58.19	1.41
11	0.71	0.72	0.02	0.73	0.70	56.92	1.02
13	0.71	0.71	0.00	0.71	0.71	147.31	1.01
14	0.40	0.40	-0.00	0.41	0.40	143.00	0.57
15	0.83	0.82	-0.03	0.85	0.81	141.40	1.17
17	0.58	0.57	-0.03	0.60	0.55	138.59	0.81
18	0.55	0.55	-0.04	0.58	0.52	137.41	0.78
20	0.24	0.24	-0.01	0.26	0.22	135.06	0.33
21	1.00	1.11	-0.18	1.17	0.96	118.66	1.49
22	0.87	0.96	-0.13	1.01	0.84	118.64	1.30

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
23	1.43	1.55	-0.29	1.61	1.38	118.71	2.11
24	1.22	1.31	-0.18	1.36	1.19	118.96	1.79
25	1.04	1.10	-0.11	1.13	1.01	119.51	1.51
27	0.53	0.55	-0.02	0.56	0.52	121.31	0.76
30	1.10	1.11	-0.04	1.13	1.09	125.16	1.56
31	0.64	0.65	-0.01	0.65	0.64	126.41	0.91
32	0.72	0.72	-0.00	0.72	0.72	126.73	1.02
33	0.79	0.78	0.01	0.79	0.78	38.45	1.11
34	0.85	0.84	0.02	0.86	0.83	39.87	1.19
35	0.64	0.64	0.01	0.65	0.63	41.54	0.91
36	1.11	1.11	0.05	1.14	1.09	43.24	1.57
37	0.52	0.52	0.01	0.53	0.51	44.92	0.74
38	0.69	0.69	0.03	0.71	0.68	46.49	0.98
39	0.76	0.77	0.04	0.80	0.74	47.84	1.09
40	0.72	0.73	0.04	0.76	0.70	49.30	1.03
41	0.65	0.66	0.04	0.69	0.63	50.80	0.93
42	1.04	1.06	0.11	1.10	1.00	52.39	1.48
43	0.99	1.02	0.10	1.06	0.95	53.78	1.42
44	1.38	1.44	0.22	1.50	1.33	55.09	1.99
47	1.69	1.84	0.38	1.91	1.65	61.79	2.50
48	2.34	2.53	0.70	2.62	2.28	61.34	3.45
49	2.08	2.23	0.52	2.31	2.02	61.06	3.05
51	0.97	1.02	0.10	1.05	0.94	59.55	1.41
52	1.16	1.21	0.13	1.25	1.13	58.34	1.68
53	1.41	1.47	0.17	1.51	1.38	57.07	2.03

FIX#	SIG X	SIG Y	SIG XY	SIG A	SIG B	THETA	SIGMA P
54	1.32	1.35	0.13	1.39	1.29	55.64	1.89
56	0.72	0.73	0.02	0.75	0.71	52.97	1.03
57	0.74	0.74	0.02	0.75	0.73	51.29	1.05
58	0.87	0.87	0.01	0.87	0.86	50.01	1.23
61	1.18	1.18	-0.06	1.21	1.16	136.12	1.67
62	0.28	0.28	-0.00	0.29	0.28	134.85	0.40
63	0.76	0.76	-0.04	0.79	0.74	133.41	1.08
67	1.14	1.18	-0.18	1.25	1.09	128.73	1.64
69	0.82	0.86	-0.12	0.92	0.77	127.53	1.19
70	0.32	0.34	-0.02	0.37	0.31	126.80	0.47
71	0.45	0.49	-0.04	0.53	0.43	125.90	0.67
73	0.83	0.90	-0.16	0.99	0.78	124.70	1.23
74	0.94	1.03	-0.22	1.13	0.88	124.25	1.40
75	0.92	1.00	-0.22	1.11	0.85	124.50	1.36

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N/MOA
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N/MOA2
National Ocean Service, NOAA
439 West York Street
Norfolk, VA 23510

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2622 Ann Street
Philadelphia, PA 19134

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NOAA Ship RAINIER
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NORWAY

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